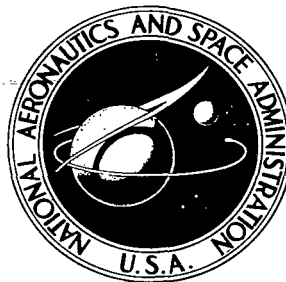


NASA TECHNICAL NOTE



NASA TN D-4756

C.1

NASA TN D-4756



LOAN COPY: RETURN TO  
AFWL (WLIL-2)  
KIRTLAND AFB, N MEX

# A MANUAL NAVIGATION METHOD FOR EVALUATING TRANSFER ORBITS FOR LUNAR LANDINGS

*by G. Kimball Miller, Jr.*  
*Langley Research Center*  
*Langley Station, Hampton, Va.*



0131256

NASA TN D-4756

A MANUAL NAVIGATION METHOD FOR EVALUATING  
TRANSFER ORBITS FOR LUNAR LANDINGS

By G. Kimball Miller, Jr.

Langley Research Center  
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information  
Springfield, Virginia 22151 - CFSTI price \$3.00

# A MANUAL NAVIGATION METHOD FOR EVALUATING TRANSFER ORBITS FOR LUNAR LANDINGS

By G. Kimball Miller, Jr.  
Langley Research Center

## SUMMARY

An analytical investigation has been made of a manual navigation method which permits an astronaut in the lunar-landing module to evaluate his orbit as he transfers from an 80-nautical-mile (148.16-kilometer) altitude circular orbit to a point approximately 50 000 feet (15 240 meters) above the lunar surface. The method can also be used to predict the altitude and velocity components at the point of nominal powered-descent initiation. When the local horizontal is known, the method requires only the astronaut's use of a sextant to measure the elevation angle to the command module and the depression angle to the lunar horizon relative to the local horizontal at a single specified measurement point and four simple guidance charts.

The navigation method was evaluated for various off-nominal transfer orbits and measurement errors. It was found that the accuracy of the method was enhanced by choosing the measurement point located as late as possible in the transfer orbit. With the measurement point located 40° prior to the point of nominal powered-descent initiation, the basic navigation method generally predicts the altitude of the landing module at pericenter and at the point of powered-descent initiation to within 2500 feet (762.0 meters). The corresponding predicted velocity components at nominal powered-descent initiation are generally within  $\pm 4$  feet per second (1.22 meters per second) of the actual values. The prediction errors experienced under the influence of selected measurement errors (including an optical determination of the local horizontal) are approximately double those of the basic method.

## INTRODUCTION

The navigation and guidance operations in Apollo space missions will normally be accomplished through the use of automatic computing equipment to process various onboard and earth-based measurements. It is desirable, however, that the astronaut also have the capability to monitor and evaluate his trajectory by means of navigation and guidance information obtained independently of the automatic system.

A method that can provide manual navigation capability for one phase of the Apollo mission is developed herein. This report is concerned with the Hohmann transfer of the landing module from the 80-nautical-mile (148.16-kilometer) altitude circular parking orbit of the command module to the 50 000-foot (14 240-meter) altitude pericenter at which the powered-descent maneuver is nominally initiated. The proposed method will enable the astronaut performing the transfer to determine in advance whether the pericenter altitude of his orbit is safe and also to predict the altitude and velocity components of the landing module at the nominal longitude of powered-descent initiation. The method requires two angular measurements and the astronaut's use of four guidance charts.

## SYMBOLS

Calculations for this investigation were made in U.S. Customary Units but are also given parenthetically in the International System of Units (SI). (See ref. 1.)

A	angle between landing-module local horizontal and line of sight to specified star, degrees
B	angle between line of sight to specified star and line of sight to lunar landmark, degrees
D	angle between landing-module local horizontal and line of sight to lunar horizon, referred to as depression angle, degrees
E	angle between landing-module local horizontal and line of sight to orbiting command module, referred to as elevation angle, degrees
$f_a$	true anomaly of apocenter, $180^\circ$
$f_F$	true anomaly of nominal point of powered-descent initiation, degrees
$f_\lambda$	true anomaly of landing module when prediction measurements are made, degrees
$g_m$	acceleration at lunar surface due to gravitational attraction, 5.32 feet/second <sup>2</sup> (1.62 meters/second <sup>2</sup> )
h	altitude above lunar surface, feet (meters)

$h_{\lambda}$	altitude above lunar surface at measurement point, feet (meters)
$P$	semilatus rectum of transfer orbit, feet (meters)
$R_m$	radius of moon, 5 702 000 feet (1 737 969.6 meters)
$r$	radial distance from center of moon ( $R_m + h$ ), feet (meters)
$\dot{r}$	landing-module radial velocity component, feet/second (meters/second)
$r\dot{\theta}$	landing-module circumferential velocity component, feet/second (meters/second)
$r_{\lambda}$	radial distance from center of moon at measurement point, feet (meters)
$T_{LM}$	orbital period of landing-module transfer orbit, seconds
$T_0$	orbital period of command-module circular orbit, seconds
$t(f_{\lambda})$	time to reach $f_{\lambda}$ , seconds
$t(\theta_{\lambda})$	time from measurement point to pericenter ( $T_{LM} - t(f_{\lambda})$ ), seconds
$\gamma$	angle between flight path and local horizontal, degrees
$\epsilon$	eccentricity of landing-module transfer orbit
$\theta_s$	selenocentric separation angle between landing module and command module, radians or degrees
$\theta_{\lambda}$	selenocentric angle from pericenter to landing module when prediction measurements are made, $360^{\circ} - f_{\lambda}$ , radians or degrees
$\lambda$	longitude of landing module when navigation measurements are made, degrees

**Subscripts:**

$a$	apocenter conditions
$F$	conditions at longitude of nominal powered-descent initiation

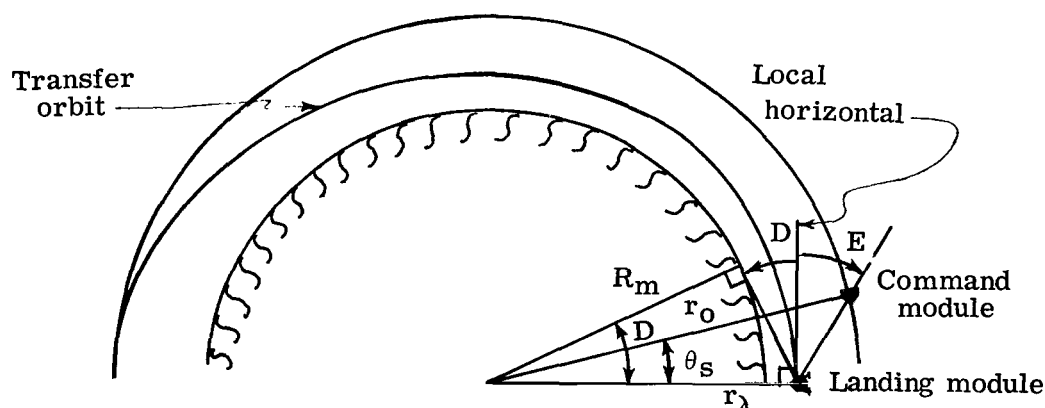
- i conditions just after transfer initiation
- o refers to circular orbit of command module
- p pericenter conditions

A delta ( $\Delta$ ) preceding a variable indicates a change in that variable from the nominal; for example,  $\Delta \dot{r} = \dot{r} - \dot{r}_{\text{nominal}}$ . A dot over a variable indicates a derivative of that variable with respect to time.

## ANALYSIS

### Basic Assumptions

The command module is assumed to be in an 80-nautical-mile (148.16-kilometer) altitude circular orbit and the Hohmann transfer orbit is assumed to be in the plane of the circular orbit of the command module. Because the landing module is only in the transfer orbit a short time, the higher order harmonics of the lunar gravitational potential may be neglected and the transfer orbit may be described with the use of Kepler's equations. With Kepler's equations, knowledge of the radial distance of the landing module from the lunar center and the true anomaly of the landing module at two points in the orbit is all that is required to determine any third point in the transfer orbit. The altitude at any point in the orbit can be determined by measuring the angle  $D$  between the landing-module local horizontal and the line of sight to the lunar horizon as shown in sketch (a):



Sketch (a)

The relationship between depression angle  $D$  and the altitude above a spherical lunar surface is given by

$$D = \cos^{-1} \left( \frac{R_m}{R_m + h_\lambda} \right) \quad (1)$$

and is presented in figure 1. The elevation angle  $E$  between the landing-module local horizontal and the line of sight to the command module is given by

$$E = \tan^{-1} \left( \frac{r_o \cos \theta_s - r_\lambda}{r_o \sin \theta_s} \right) \quad (2)$$

where the selenocentric separation angle  $\theta_s$  is a function of the selenocentric angle from pericenter  $\theta_\lambda$  and the time to reach pericenter  $t(\theta_\lambda)$ :

$$\theta_s = \frac{2\pi}{T_o} \left[ t(\theta_\lambda) + \frac{T_o - T_{LM}}{2} \right] - \theta_\lambda \quad (3)$$

Thus,  $E$  is a function of  $\theta_\lambda$  and  $h_\lambda$ . However, it can be shown that a change in altitude  $h_\lambda$  has very little effect on elevation angle  $E$  for a given position in the orbit  $\theta_\lambda$ . (See appendix A.) Thus,  $E$  may be considered a function of  $\theta_\lambda$  only. The relationship between  $E$  and  $\theta_\lambda$  is presented in figure 2 for a nominal Hohmann transfer from an 80-nautical-mile (148.16-kilometer) circular orbit to a pericenter altitude of 50 000 feet (15 240 meters). Off-nominal transfer orbits from 80 nautical miles (148.16 kilometers) will have essentially the same relationship between elevation angle and selenocentric position angle. Thus, if the astronaut can measure the elevation angle  $E$  at some point during his transfer, he can use a chart such as figure 2 to determine his position on the orbit.

The use of  $D$  and  $E$  measurements in conjunction with charts similar to figures 1 and 2 to define a point  $(h_\lambda, \theta_\lambda)$  on the transfer orbit is the basis of the simplified navigation method.

### Formulation of Navigation Method

In the event that the landing-module local horizontal is known (the unknown case and various operational aspects are considered in appendix B), the method requires only the measurement of the elevation angle  $E$  and the depression angle  $D$  to compute pericenter altitude. However, in order to predict the landing-module conditions at the point of nominal powered-descent initiation, the longitude of the landing module at the measurement point is required. This information is acquired through the use of a specified, readily recognizable landmark located at longitude  $\lambda$ . (See fig. 3.)

Prior to reaching the measurement point, the astronaut orients the landing module perpendicular to the local horizontal of the specified measurement point and observes the

landmark. When the landing module is directly over the landmark, as determined through the use of a sighting device in the overhead window of the landing module, the astronaut knows he has reached the measurement point. The astronaut then uses the sextant to measure the elevation angle to the command module and the depression angle to the lunar horizon. (Because  $E$  and  $D$  vary with time, some updating or rather postdating of the measurements is required to obtain the values simultaneously at the measurement point.) The astronaut can then use charts corresponding to figures 1 and 2 to obtain his selenocentric position  $\theta_\lambda$  and altitude  $h_\lambda$ , and thus to define a point in the transfer orbit. He can define a second point in the transfer orbit in the same manner; the second point is all that is required to solve ordinary Keplerian equations for a third point.

Because this investigation is concerned with Hohmann transfer trajectories, a simpler method which should be of comparable accuracy for off-nominal transfer orbits of interest is used. In the proposed method, only one point is defined by measuring  $E$  and  $D$ . The other point required for the solution of Kepler's equations is defined by assuming that the apocenter altitude  $h_a$  of the transfer orbit is equal to the nominal orbit altitude of the command module, 80 nautical miles (148.16 kilometers). The true anomaly of apocenter is  $180^\circ$ . The second point is thus defined by an  $h_a$  of 80 nautical miles (148.16 kilometers) and  $f_a = 180^\circ$ .

If  $h_a$  and  $f_a$  are known and  $h_\lambda$  and  $\theta_\lambda$  have been determined from the measured angles  $D$  and  $E$  through the use of figures 1 and 2, the astronaut can predict his pericenter altitude by solving the following Keplerian equation:

$$h_p = r_p - R_m = \frac{P}{1 + \epsilon} - R_m \quad (4)$$

where

$$P = r_\lambda (1 + \epsilon \cos \theta_\lambda) \quad (5)$$

and

$$\epsilon = \frac{r_a - r_\lambda}{r_\lambda \cos \theta_\lambda - r_a \cos f_a} \quad (6)$$

If the astronaut finds that the pericenter altitude of his transfer orbit is safe, he can determine his orbital parameters at the nominal point of powered-descent thrust initiation (arbitrarily chosen as  $0^\circ$  longitude in this investigation) by solving the following equations:

$$f_F = \lambda - \theta_\lambda \quad (7)$$

$$h_F = r_F - R_m = \frac{P}{1 + \epsilon \cos f_F} - R_m \quad (8)$$



$$(r\dot{\theta})_F = \frac{R_m}{r_F} \sqrt{g_m P} \quad (9)$$

$$\dot{r}_F = (r\dot{\theta})_F \tan \gamma_F \quad (10)$$

where

$$\gamma_F = \tan^{-1} \frac{\epsilon \sin f_F}{1 + \epsilon \cos f_F} \quad (11)$$

The astronaut could then use the predicted values of his orbital parameters at the point of nominal powered-descent initiation in conjunction with a backup landing procedure to attain the nominal landing site.

#### Evaluation of Navigation Method

An electronic digital computer was used to compute the actual values of  $E$  and  $D$  when the landing module reached the measurement point in various off-nominal transfer orbits. The resulting values of  $E$  and  $D$  were then entered into figures 1 and 2 to obtain  $\theta_\lambda$  and  $h_\lambda$ , respectively. The predicted values of  $h_p$  and the orbital parameters at the nominal point of powered-descent initiation were then computed from equations (4) to (11) by using an ordinary desk calculator. These predicted values were then compared with the actual values of the orbital parameters generated by the digital computer solving Keplerian equations. The off-nominal orbits used to evaluate the manual navigation method are the result of various deviations in the velocity components and altitude from the nominal at the Hohmann initiation point. Various combinations of  $\pm 10$ ,  $\pm 5$ , and 0 feet per second ( $\pm 3.048$ ,  $\pm 1.524$ , and 0 meters per second) deviations in the velocity components and  $\pm 1$ -nautical-mile ( $\pm 1.852$ -kilometer) deviation in circular-orbit altitude were considered. Also included was the effect of the location  $\lambda$  of the point at which the navigation measurements were made. The measurement point should be as near pericenter as possible because for a given accuracy in defining the measurement point, the accuracy in predicting conditions at pericenter is inversely proportional to the distance between pericenter and the point at which the prediction is made. Measurement points located  $40^\circ$  and  $80^\circ$  east of the nominal point of powered-descent initiation ( $\lambda = 40^\circ$  and  $\lambda = 80^\circ$ ) were used in evaluating the navigation method. These measurement points are located approximately 46 and 33 minutes after Hohmann insertion, respectively, and 12 and 25 minutes prior to pericenter.

## DISCUSSION OF RESULTS

### Accuracy of the Method

When the off-nominal transfer orbit is initiated at the nominal altitude of 80 nautical miles (148.16 kilometers), the pericenter altitudes for the circumferential velocity deviations considered range from about 93 000 feet (28 346.4 meters) to 7500 feet (2285.0 kilometers). (See fig. 4(a).) For the range of off-nominal velocity components considered, the manual navigation method always predicts the pericenter altitude to within 2500 feet (762.0 meters) when the predictions are based on measurements made at  $\lambda = 40^\circ$ . The prediction errors increase to as much as 9000 feet (2743.2 meters) when the measurements are made at  $\lambda = 80^\circ$ . Deviation of  $\pm 1$  nautical mile (1.852 kilometers) in the altitude of the circular orbit of the command module (figs. 4(b) and 4(c)) have little effect on the accuracy of the navigation method. The prediction errors remain generally less than 2500 feet (762.0 meters) and 9000 feet (2743.2 meters) when the measurements are made at  $\lambda = 40^\circ$  and  $\lambda = 80^\circ$ , respectively.

The performance of the manual navigation method in predicting what the transfer orbit altitude will be when the landing module reaches the point of nominal powered-descent initiation (fig. 5) is approximately the same as was the case for pericenter-altitude prediction. The prediction errors are again generally less than 2500 feet (762.0 meters) and 9000 feet (2743.2 meters) when the measurements are made at  $\lambda = 40^\circ$  and  $\lambda = 80^\circ$ , respectively.

For the off-nominal orbits considered in this investigation, the circumferential velocity at the point of nominal powered-descent initiation ranges from approximately 5550 feet per second (1691.6 meters per second) to 5615 feet per second (1711.4 meters per second). (See fig. 6.) When the measurements are made at  $\lambda = 40^\circ$ , the manual navigation method generally predicts the circumferential velocity to within 2 feet per second (0.61 meter per second). (See figs. 6(a) and 6(c).) However, when the altitude of the circular orbit of the command module is 79 nautical miles (146.31 kilometers), the prediction error increases (fig. 6(b)) to as much as 4 feet per second (1.22 meters per second). When the measurements are made at  $\lambda = 80^\circ$ , the manual navigation method predicts the circumferential velocity to within 6 feet per second (1.83 meters per second) for all the off-nominal transfer orbits considered.

The radial velocity at the point of nominal powered-descent initiation ranges from about 10 feet per second (3.05 meters per second) to -10 feet per second (-3.05 meters per second) (fig. 7) for the off-nominal transfer orbits considered in this investigation. The manual navigation method predicts the radial velocity to within 4 feet per second (1.22 meters per second) and 5 feet per second (1.52 meters per second) when the measurements are made at  $\lambda = 40^\circ$  and  $\lambda = 80^\circ$ , respectively.

This discussion is concerned with the accuracy of the basic manual navigation method and does not include the effects of errors in the measurement of the elevation angle  $E$  and depression angle  $D$ . An error analysis is included in appendix B which considers the effect of various sighting errors on the accuracy of the manual navigation method when the astronaut is required to determine the landing-module local horizontal optically by use of sextant sightings on a specified star and landmark. The error analysis indicated that measurement errors of  $\pm 0.1^\circ$  in elevation angle and  $\pm 0.2^\circ$  in depression angle approximately double the prediction errors of the basic method. For example, at a  $\lambda$  of  $40^\circ$ , the navigation method, under the influence of these measurement errors, generally predicts the landing-module altitude at pericenter and at the point of nominal powered-descent initiation to within 6000 feet (1828.8 meters) and the velocity components at nominal powered-descent initiation to within  $\pm 6$  feet per second ( $\pm 1.83$  meters per second). Thus, the advantage in performing the navigation measurements as late as possible in the transfer orbit is emphasized, because for measurements made at  $\lambda = 40^\circ$ , the predicted values of the navigation method under the influence of measurement errors are still more accurate than those of the basic method based on measurements made at  $\lambda = 80^\circ$ .

#### Guidance Charts

In order to enhance the accuracy of the navigation method, it is desirable to perform the measurements late in the transfer orbit. However, when the measurements are made at  $\lambda = 40^\circ$ , the astronaut has less than 15 minutes in which to perform the computations required by the method before the landing module reaches pericenter. This amount of time is inadequate for performing the required computations on a desk-type calculator. Consequently, the navigation equation may be solved graphically through the use of the guidance charts 1 to 4, for measurements made at  $\lambda = 40^\circ$ . It should be noted that the guidance charts are presented with low resolution for conciseness and the actual guidance charts used by the astronauts would possess a considerably higher degree of resolution.

Thus, the manual navigation method requires only the astronaut's use of a sextant to measure the elevation angle  $E$  to the command module and the depression angle  $D$  to the lunar horizon and the use of four simple guidance charts. The navigation method provides the astronaut with a means for determining whether the pericenter altitude of the landing module is safe (chart 1). If pericenter is safe, the altitude and velocity components at the point of nominal powered-descent initiation can be determined from the three remaining charts. When the altitude and velocity components at nominal powered-descent initiation are known, the pilot could use a backup landing procedure to attain the nominal landing site when the primary guidance and navigation system are inoperative.

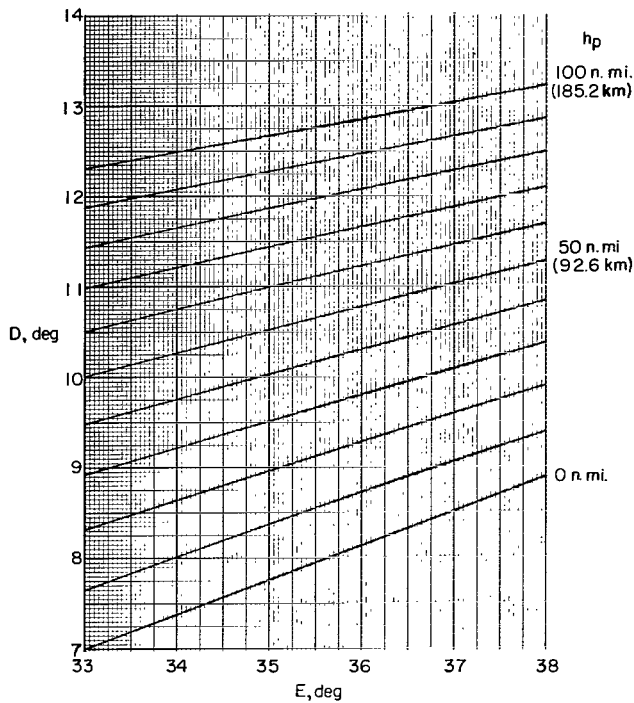


Chart 1.- Pericenter altitude.

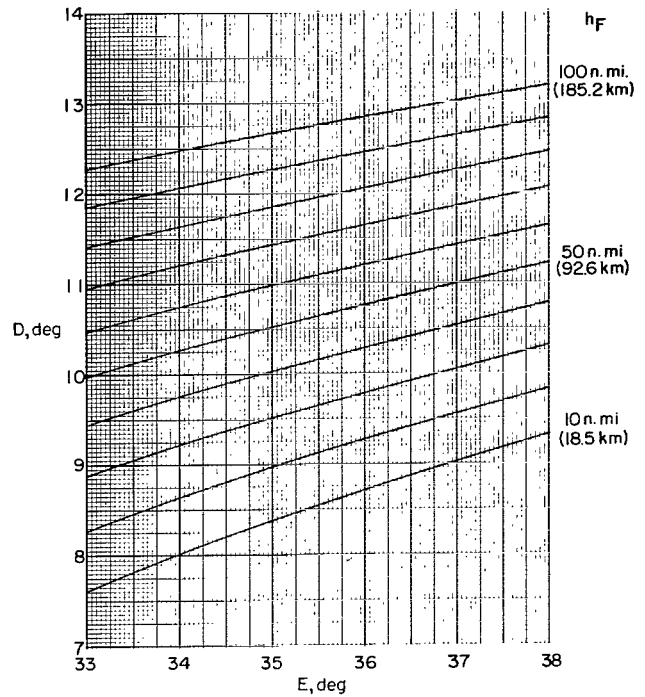


Chart 2.- Altitude at nominal point of powered descent ( $0^\circ$  East).

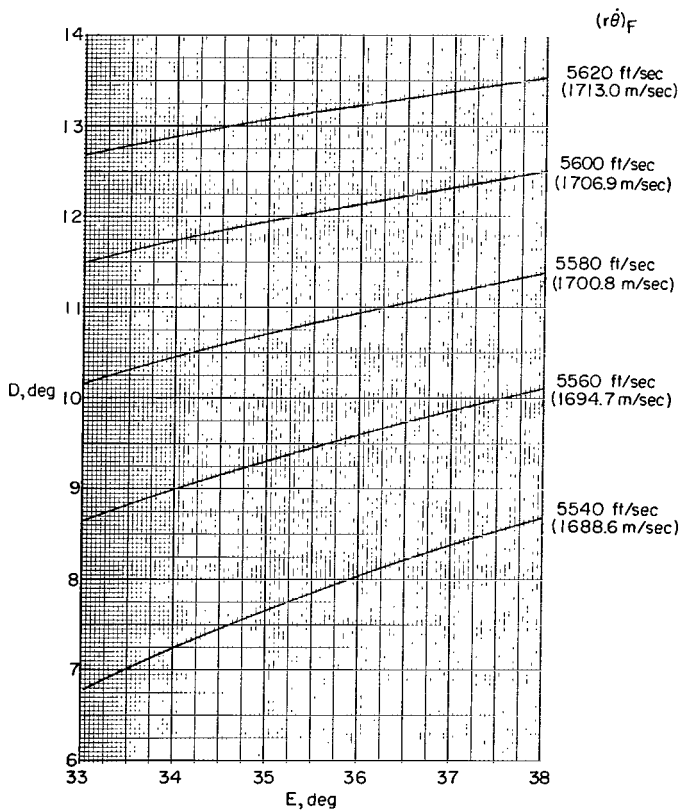


Chart 3.- Radial velocity at nominal point of powered descent ( $0^\circ$  East).

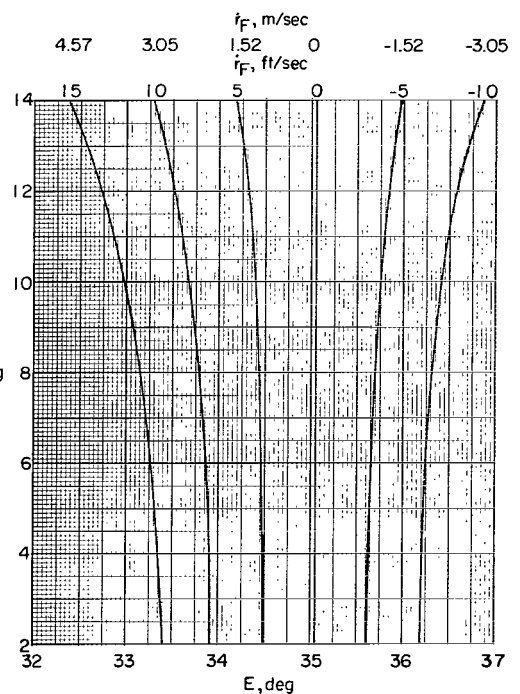


Chart 4.- Circumferential velocity at nominal point of powered descent ( $0^\circ$  East).

## CONCLUDING REMARKS

An analytical investigation has been made of a manual navigation method which will permit an astronaut to predict the pericenter altitude of the landing module as it moves in its transfer orbit from the circular orbit of the command module to the point of powered-descent initiation. The method can also be used to predict the altitude and velocity components of the landing module at the point of nominal powered-descent initiation. When the local horizontal is known, the manual navigation method requires only the astronaut's use of a sextant to measure the elevation angle to the command module and the depression angle to the lunar horizon at a single measurement point (which was defined by sighting on a specified lunar landmark) and four simple guidance charts.

The navigation method was evaluated with the measurement point located  $40^{\circ}$  and  $80^{\circ}$  prior to the point of nominal powered-descent initiation for a range of off-nominal transfer orbits generated by using deviations in the velocity components at Hohmann transfer initiation of  $\pm 5$  and  $\pm 10$  feet per second ( $\pm 1.52$  and  $\pm 3.05$  meters per second) and deviations in altitude of  $\pm 1$  nautical mile ( $\pm 1.852$  kilometers). It was found that the accuracy of the method is enhanced when the measurement point is located late in the transfer orbit. For the errors considered and with the measurements made at a point  $40^{\circ}$  prior to the point of nominal powered-descent initiation, the basic navigation method generally predicts the altitude of the landing module at pericenter and at the point of nominal powered-descent initiation to within 2500 feet (762.0 meters). The corresponding predicted values of the velocity components at nominal powered-descent initiation are within  $\pm 4$  feet per second ( $\pm 1.22$  meters per second) of the actual values.

The prediction errors experienced when the landing-module local horizontal is determined optically by use of a specified star and landmark and with measurement uncertainties of  $\pm 0.1^{\circ}$  in elevation angle and  $\pm 0.2^{\circ}$  in depression angle are approximately double those of the basic navigation method.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., May 28, 1968,  
127-51-06-01-23.

## APPENDIX A

### VARIATION OF $E$ WITH ALTITUDE

Preliminary investigations using an electronic digital computer to generate several off-nominal transfer orbits indicated that  $\frac{\partial E}{\partial h_\lambda}$  remained very near its pericenter value for landing-module positions up to  $120^\circ$  ( $\theta_\lambda = 120^\circ$ ). Therefore, that  $E$  varies insignificantly with changes in  $h$  may be shown by considering the case when the landing module is at the pericenter of its transfer orbit. At pericenter the separation angle may be expressed as

$$\theta_s = \frac{2\pi}{T_o} \left( \frac{T_o - T_{LM}}{2} \right) \quad (A1)$$

which can be written as

$$\theta_s = \pi \left[ 1 - \left( \frac{1}{2} + \frac{r_p}{2r_o} \right)^{3/2} \right] \quad (A2)$$

Differentiating equation (2), with  $r_\lambda = r_p$ , and equation (A2) with respect to  $r_p$  and combining the results yields

$$\frac{\partial E}{\partial r_p} = \frac{1}{r_o} \left[ \frac{\frac{3\sqrt{2}}{8} \pi \left( 1 + \frac{r_p}{r_o} \right)^{1/2} \left( 1 - \frac{r_p}{r_o} \cos \theta_s \right) - \sin \theta_s}{\left( \frac{r_p}{r_o} \right)^2 - 2 \left( \frac{r_p}{r_o} \right) \cos \theta_s + 1} \right] \quad (A3)$$

For pericenter altitude of 50 000 feet (15 240 meters) and a circular orbit altitude of 80 nautical miles (148.16 kilometers),  $\frac{\partial E}{\partial r_p} \approx 0.1^\circ$  per 10 000 feet (3048 meters).

Thus, it appears that, for the region of interest, changes in  $h$  have very little effect on the elevation angle  $E$ . The basic assumption made in developing the navigation method then is that at a given position in the transfer orbit, the elevation angle  $E$  is a constant regardless of the altitude.

## APPENDIX B

### ERROR ANALYSIS

In the event that the local horizontal of the landing module is not known, it must be determined. One method for obtaining the local horizontal at a known longitude  $\lambda$  is used in this section. A readily recognizable landmark located at longitude  $\lambda$  and a star are specified so that the angle  $B$  between the lines of sight to the landmark and the star has a specified value when the landing module is directly over the landmark. (See fig. 8.) Prior to reaching the landmark, the astronaut sights on the specified star with the angle  $B$  set into the instrument. When the landmark is superimposed on the star, the astronaut will know that the landing module local horizontal is located at the specified angle  $A$  above the line of sight to the star. Having determined the location of the landing module's local horizontal at longitude  $\lambda$ , the astronaut can use the sextant to measure the angle between the lines of sight to the star and the command module and subtract the known angle  $A$  to obtain the elevation angle  $E$ . He can then measure the angle between the lines of sight to the star and the lunar horizon and add the known angle  $A$  to obtain the depression angle  $D$ . The astronaut can then use the guidance charts of the basic method to predict his pericenter altitude and, if he decides that his orbit is satisfactory, to determine the vehicle parameters at the nominal point of powered-descent initiation.

#### Operational Aspects

The orbit plane component of the angle between the lines of sight to the command module and the sun is approximately  $25^\circ$  and  $40^\circ$  at  $\lambda$  equal  $40^\circ$  and  $80^\circ$ , respectively, for a  $7^\circ$  over-the-shoulder sun angle at the nominal landing site. Thus, the sun does not present a glare problem and the command module should be visible. (See ref. 2.) Because the local horizontal, the elevation angle  $E$ , and the depression angle  $D$  all change with time, an updating method will be required to obtain the values of  $E$  and  $D$  when the landing module is directly over the landmark. The elevation and depression-angle measurements may be readily obtained through the front window of the landing module. However, sighting on the lunar landmark requires the use of the overhead window. In addition, the sextant must possess magnification commensurate with the accuracy requirements and a field of view sufficiently large to facilitate acquisition of the measurement targets. (See ref. 3.) The operational difficulties associated with the reference star and lunar landmark being slightly out of the orbit plane have not been included in this investigation. The determination of the nominal point of powered-descent initiation has not been dealt with specifically, but could be accomplished through the graphical solution of the time equation (a fifth chart) or by sighting on a specified landmark.

## APPENDIX B

### Accuracy of Navigation Method

In most navigation methods the positions at which the measurements are made are randomly selected. In the method presented, however, the location of the single measurement point is specified and thus may be chosen so that the effect of the uncertainties in the lunar figure is minimized. Lunar-figure uncertainties and sextant-astronaut sighting errors result in uncertainties in elevation angle  $\Delta E$  and depression angle  $\Delta D$ . With the local horizontal determined optically, uncertainties of  $\pm 0.1^\circ$  in  $\Delta E$  and  $\pm 0.15^\circ$  in  $\Delta D$  were assumed when the measurements were made at  $\lambda = 80^\circ$ . The corresponding uncertainty  $\Delta D$  for measurements made at  $\lambda = 40^\circ$  was  $\pm 0.2^\circ$  because the uncertainty is inversely proportional to altitude.

The error analysis includes only those combinations of  $\pm \Delta E$  and  $\pm \Delta D$  which result in the lowest predicted pericenter altitudes ( $+\Delta E$ ,  $-\Delta D$ ) and the highest predicted pericenter altitudes ( $-\Delta E$ ,  $+\Delta D$ ). The predicted pericenter altitudes for other combinations of  $\Delta E$  and  $\Delta D$  fall between these two and for brevity have been omitted. Also in the interests of brevity and because the basic method was shown to be relatively insensitive to deviations in the altitude of the circular orbit of the command module, the error analysis only considers transfer orbits initiated at 80 nautical miles (148.16 kilometers). The effect of  $+\Delta E$  and  $-\Delta D$  measurement errors on the performance of the manual navigation method is shown in figure 9 and the effect of  $-\Delta E$  and  $+\Delta D$  measurement errors is shown in figure 10 (a  $\Delta D$  error of  $0.1^\circ$  has been included for comparison because Lunar Orbiter data may result in the reduction of lunar feature uncertainties). For the transfer orbits considered, the predicted values of the pericenter altitude are generally within 6000 feet (1828.8 meters) of the actual values (figs. 9(a) and 10(a)) when the navigation measurements are made at  $\lambda = 40^\circ$ , but in some instances may miss the actual value by almost 9000 feet (2743.2 meters) (fig. 9(a)). When the measurements are made at  $\lambda = 80^\circ$ , the errors increase to the point where the predicted values may be in error by as much as 18 000 feet (5486.4 meters) (fig. 9(a)). In general, the prediction errors are lowest for the lowest pericenter altitudes. The errors in predicting the altitude of the landing module at the point of nominal powered-descent initiation (figs. 9(b) and 10(b)) are approximately the same as the errors in predicting the pericenter altitude.

The predicted values of the circumferential velocity of the landing module at the point of nominal powered-descent initiation always falls within  $\pm 6$  feet per second ( $\pm 1.83$  meters per second) of the actual values (figs. 9(c) and 10(c)) when the navigation measurements are made at  $\lambda = 40^\circ$ , but only fall within about  $\pm 12$  feet per second ( $\pm 3.66$  meters per second) of the actual values when the measurements are made at  $\lambda = 80^\circ$ . The errors involved in predicting the radial velocity of the landing module at the point of nominal powered-descent initiation (figs. 9(d) and 10(d)) lie between -6 feet per second (-1.83 meters per second) and 4 feet per second (1.22 meters per second) whether the navigation measurements are made at  $\lambda = 40^\circ$  or  $\lambda = 80^\circ$ .



## APPENDIX B

In summary, the effect of the measurement errors considered is to approximately double the errors of the basic navigation method. The advantage of making the navigation measurements late in the transfer orbit is emphasized by the fact that for measurements at  $\lambda = 40^\circ$ , the predicted values of the method under the influence of measurement errors are still more accurate than those made at  $\lambda = 80^\circ$  with the basic method.

## REFERENCES

1. Mechtly, E. A.: The International System of Units – Physical Constants and Conversion Factors. NASA SP-7012, 1964.
2. Baker, C. A., ed.: Visual Capabilities in the Space Environment. Pergamon Press, Inc., c.1965.
3. Smith, Donald W.: The Hand-Held Sextant: Results From Gemini XII and Flight Simulator Experiments. Paper No. 67-775, Amer. Inst. Aeronaut. Astronaut., Oct. 1967.

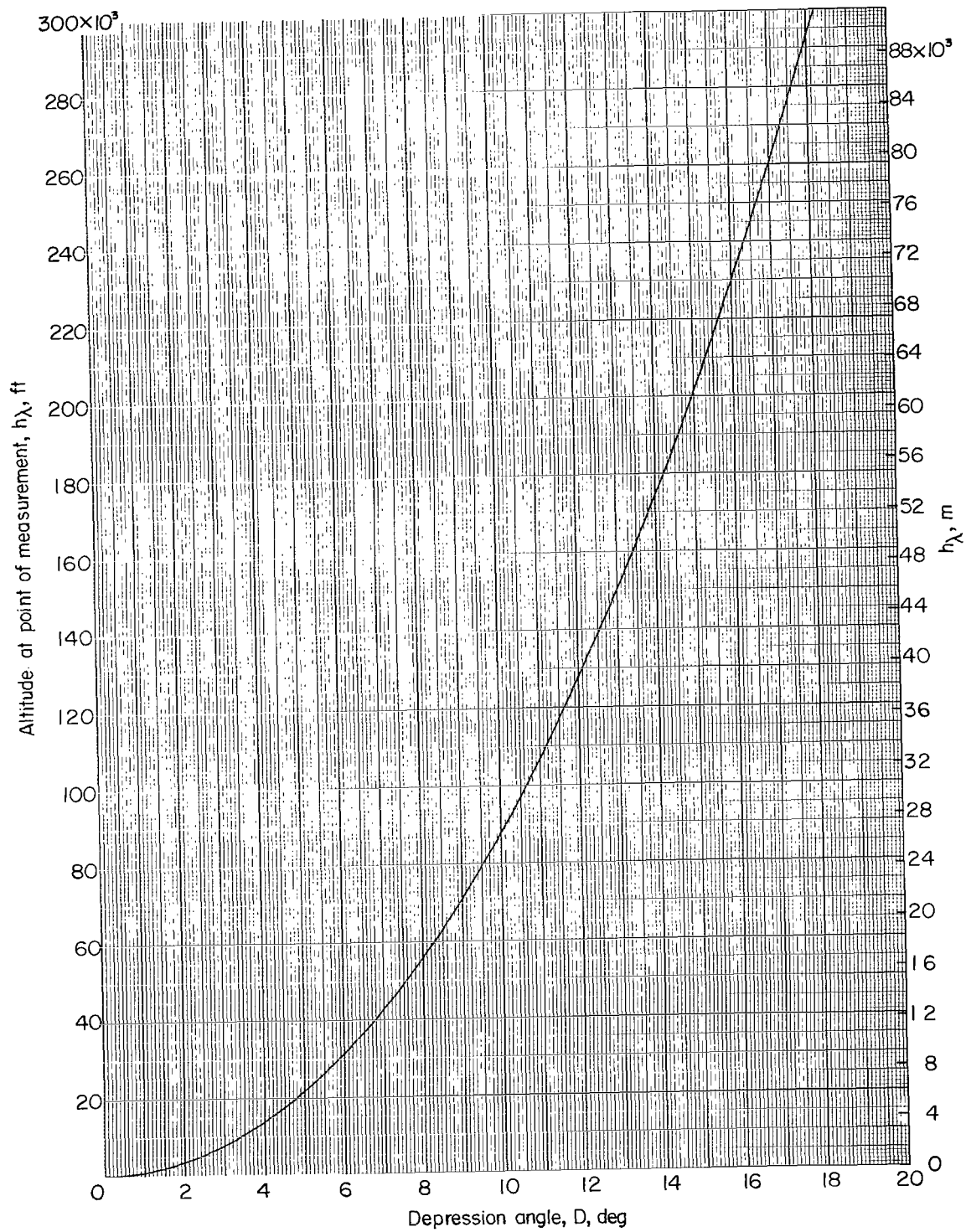


Figure 1.- Relationship between altitude and depression angle.

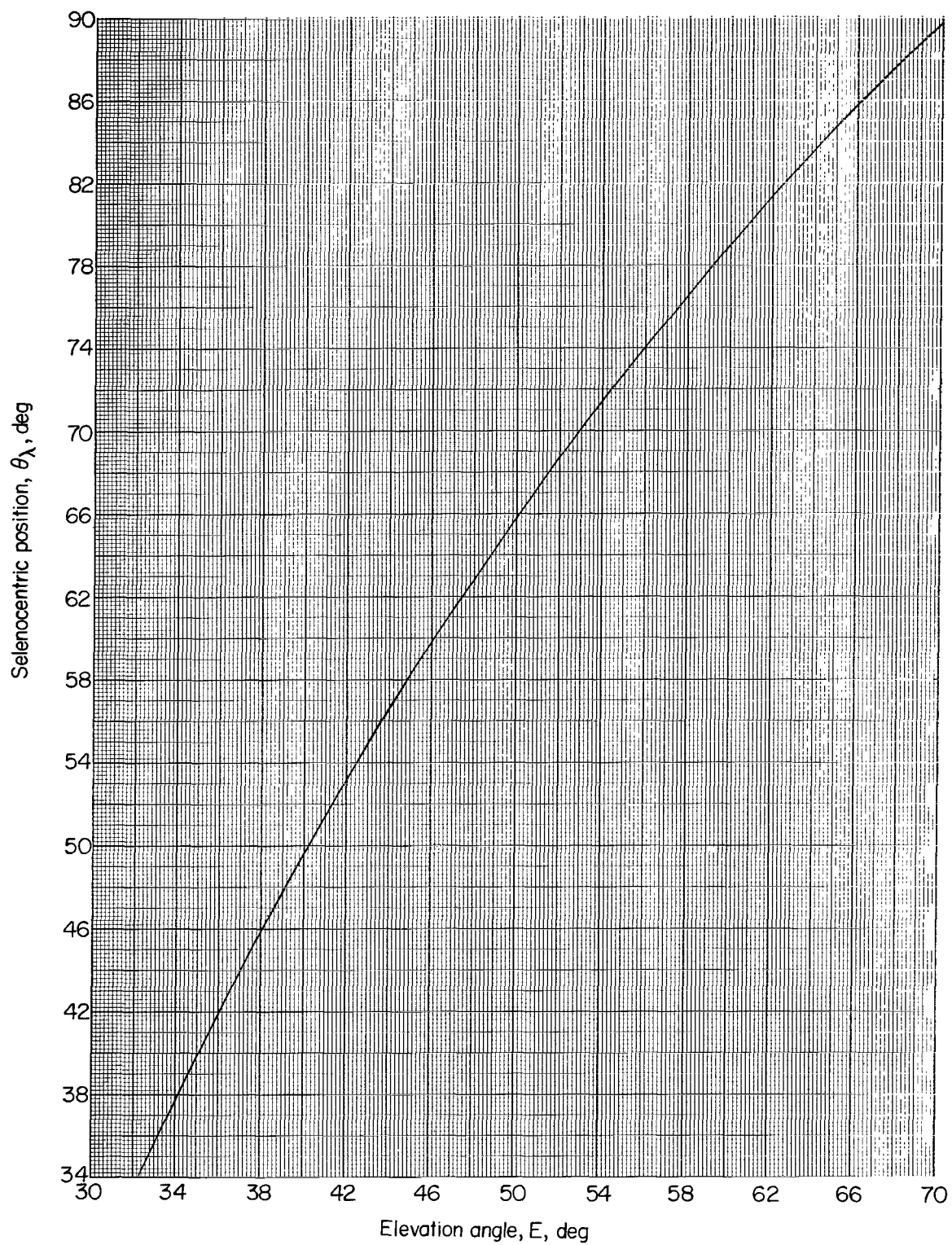


Figure 2.- Relationship between selenocentric position and elevation angle.

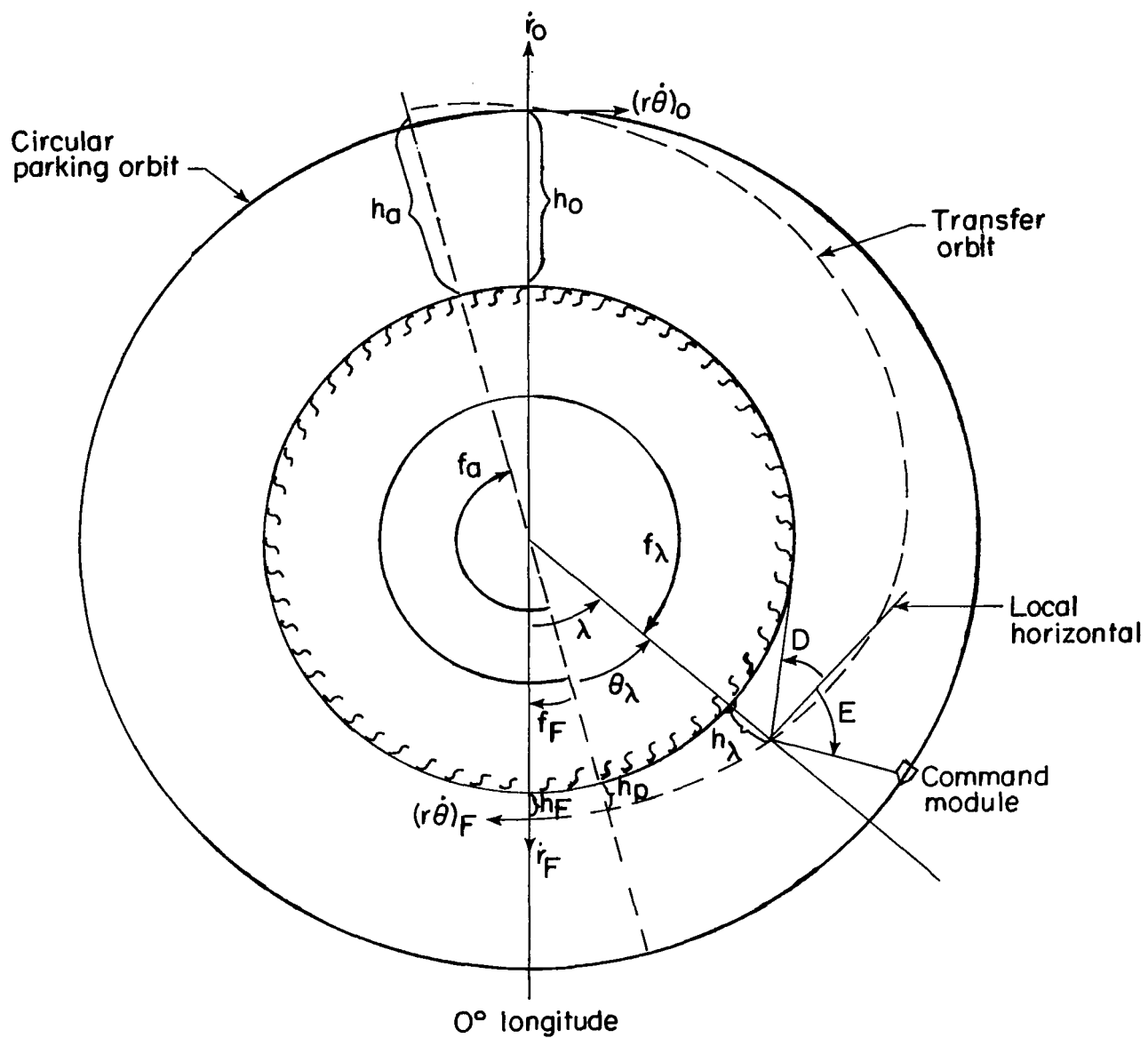
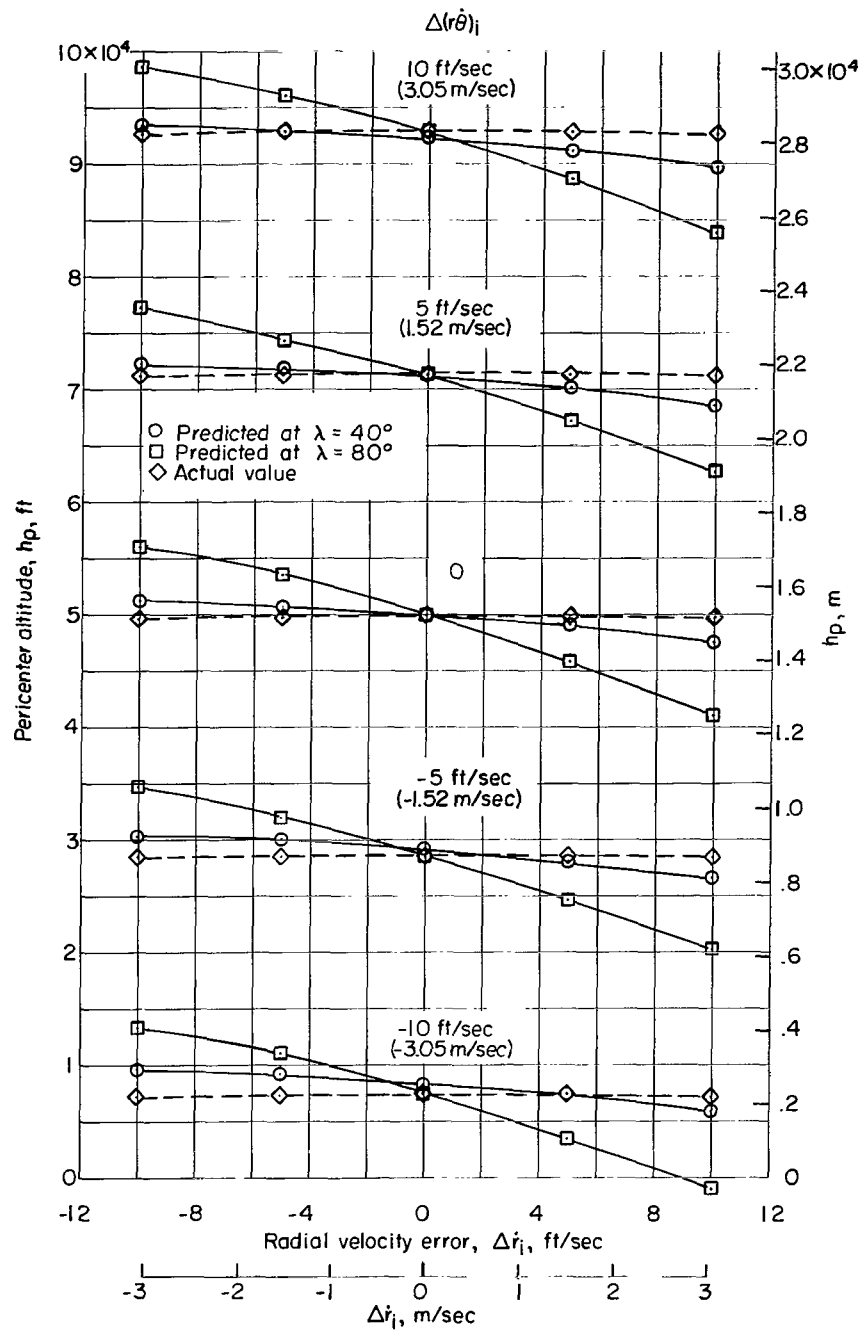
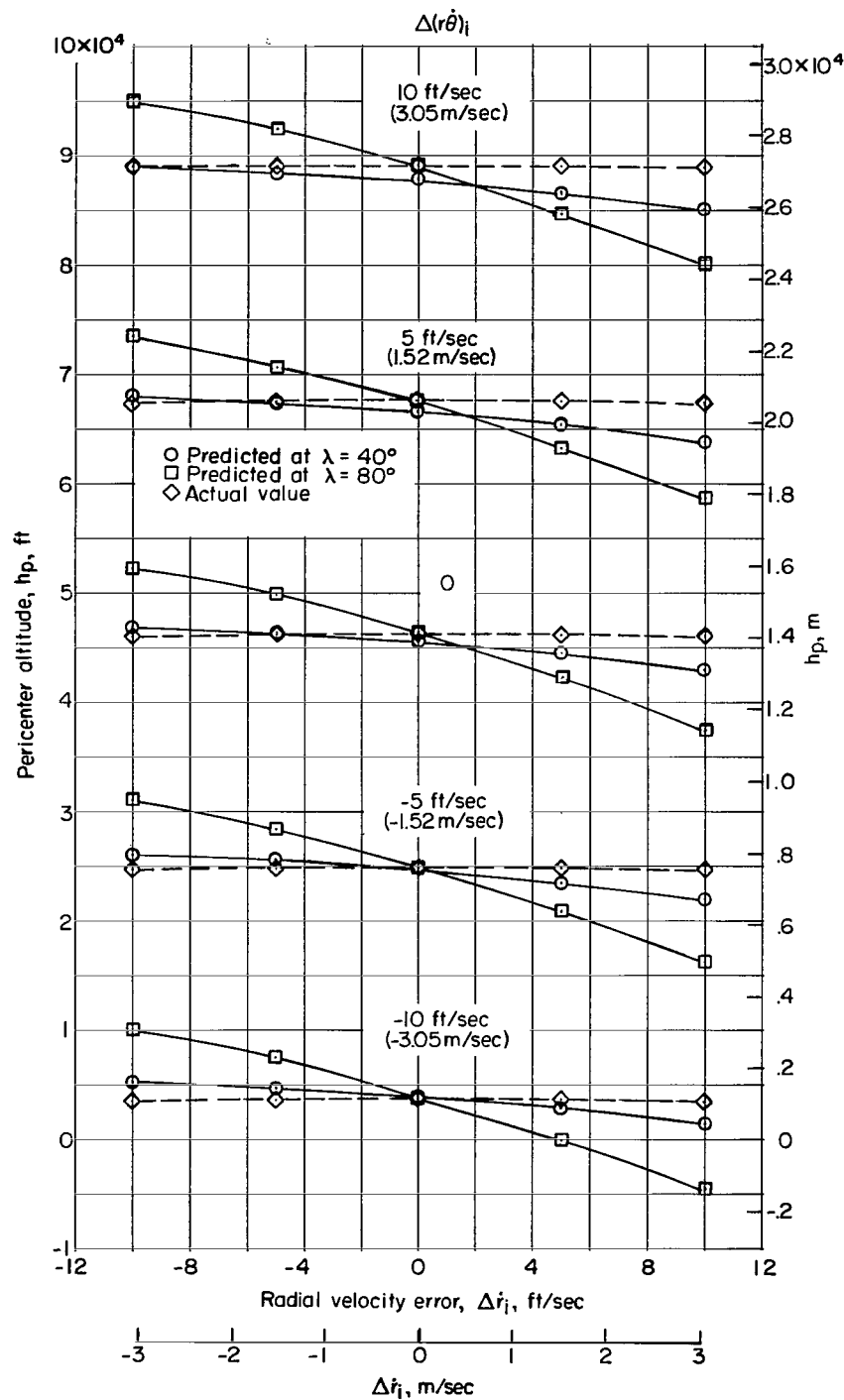


Figure 3.- Illustration of basic navigation.



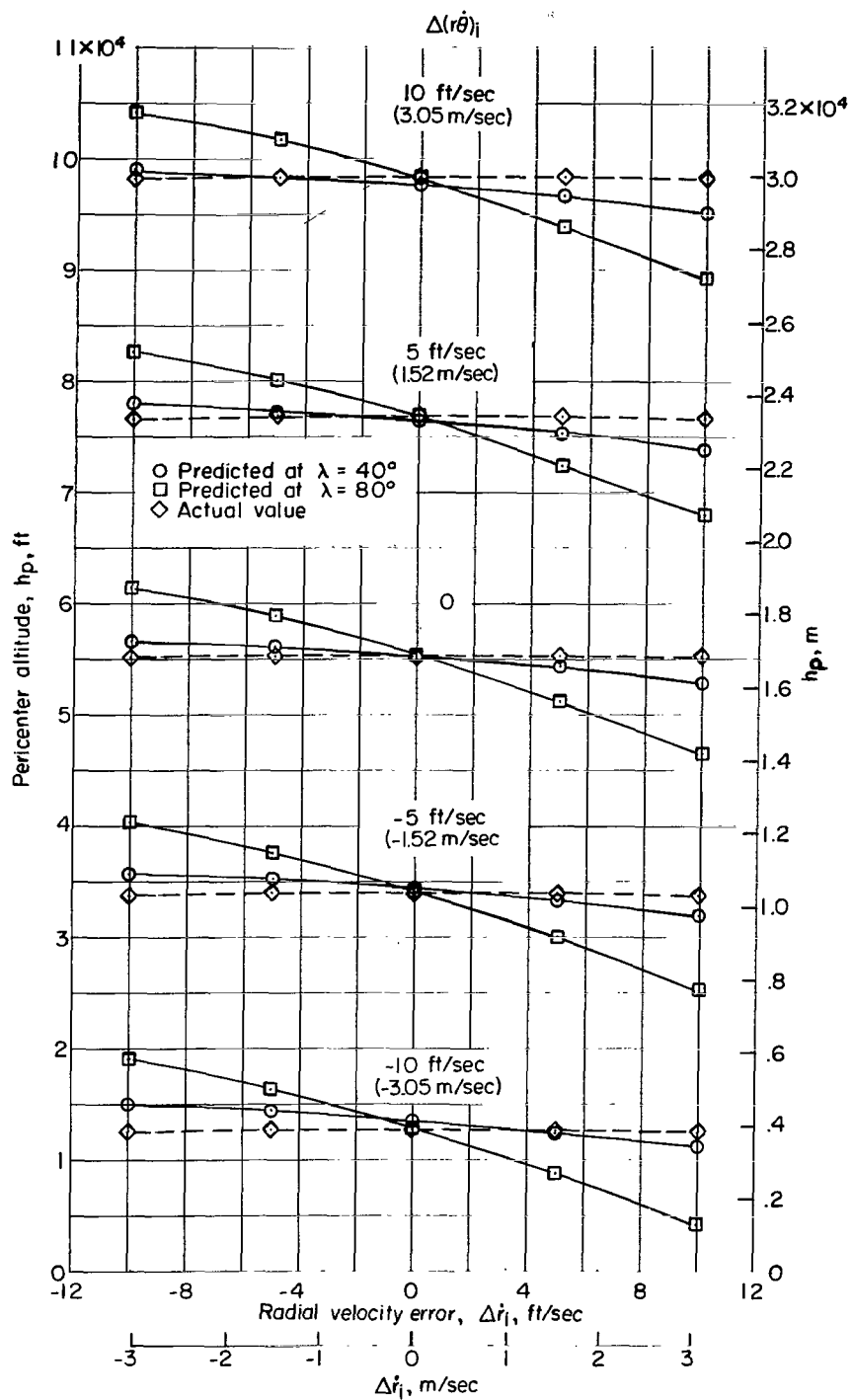
(a) Command module in 80-nautical-mile (148.16-kilometer) altitude circular orbit ( $\Delta h_0 = 0$ ).

Figure 4.- Pericenter altitude for various velocity errors at Hohmann insertion.



(b) Command module in 79-nautical-mile (146,31-kilometer) altitude circular orbit ( $\Delta h_0 = -1$  nautical mile or -1.852 kilometers).

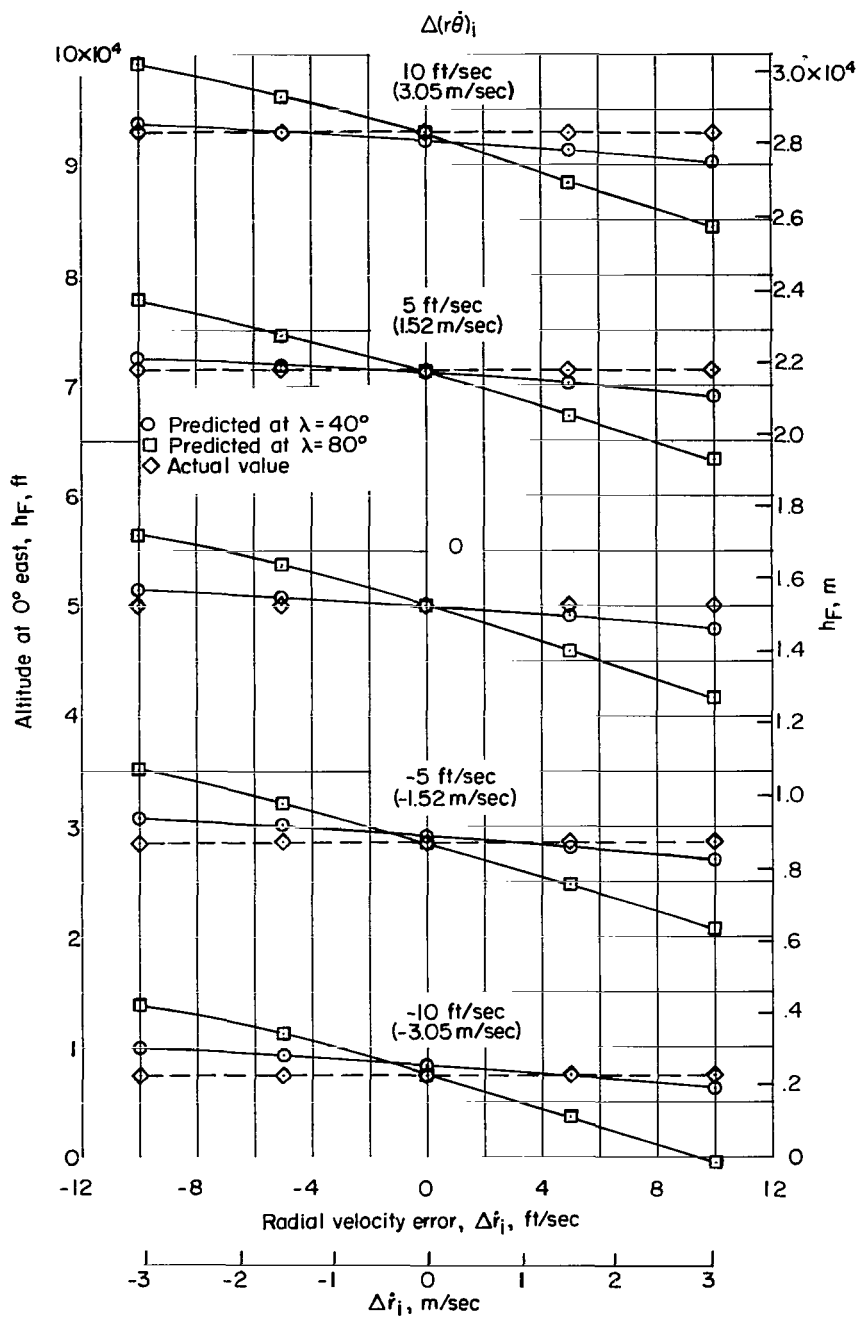
Figure 4.- Continued.



(c) Command module in 81-nautical mile (150.01-kilometer) altitude circular orbit ( $\Delta h_0 = 1$  nautical mile or 1.852 kilometers).

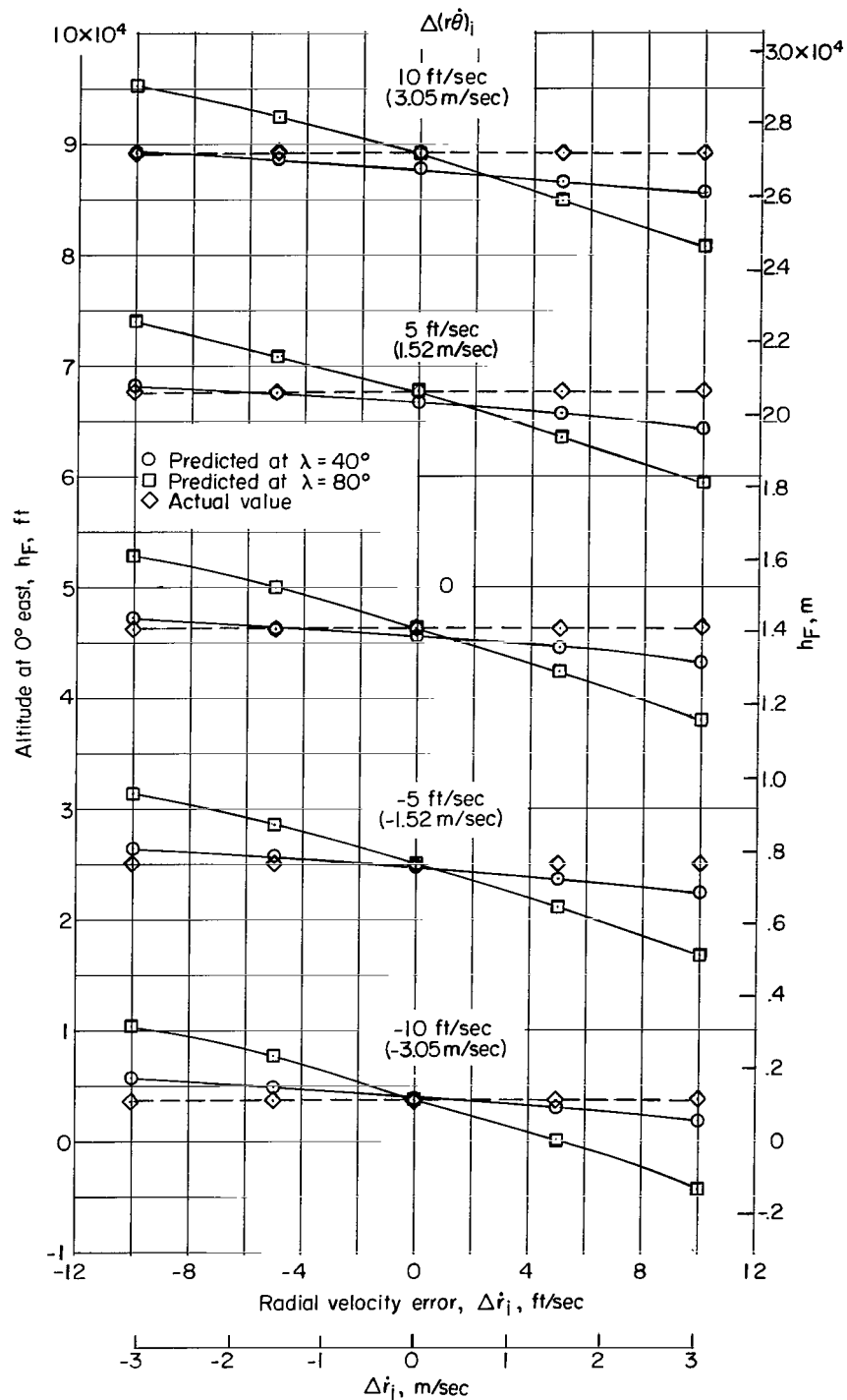
Figure 4.- Concluded.





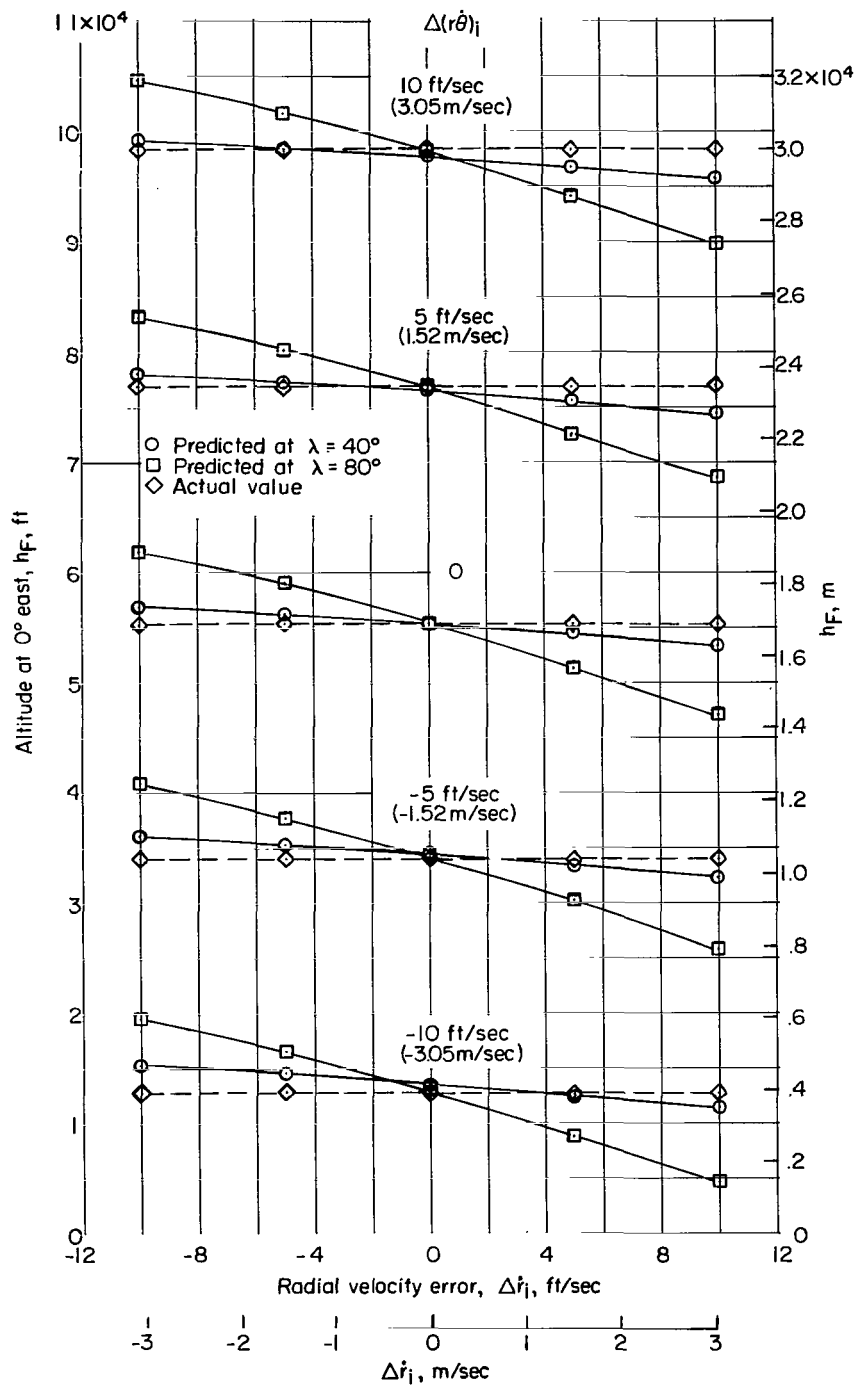
(a) Command module in 80-nautical-mile (148.16-kilometer) altitude circular orbit ( $\Delta h_0 = 0$ ).

Figure 5.- Landing module altitude at point of nominal powered-descent initiation ( $0^\circ$  East longitude) for various velocity errors at Hohmann insertion.



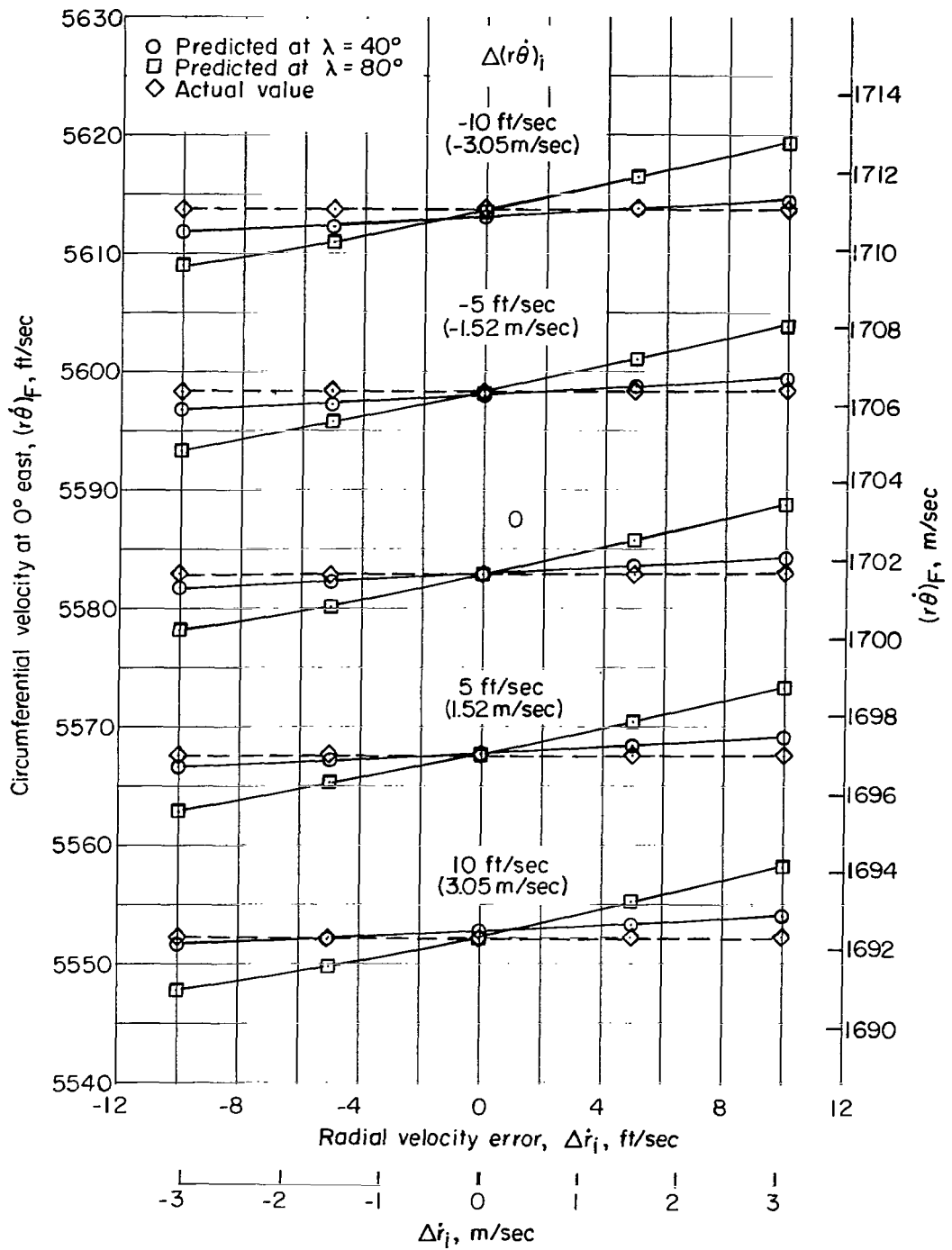
(b) Command module in 79-nautical-mile (146.31-kilometer) altitude circular orbit ( $\Delta h_0 = -1$  nautical mile or -1.852 kilometers).

Figure 5.- Continued.



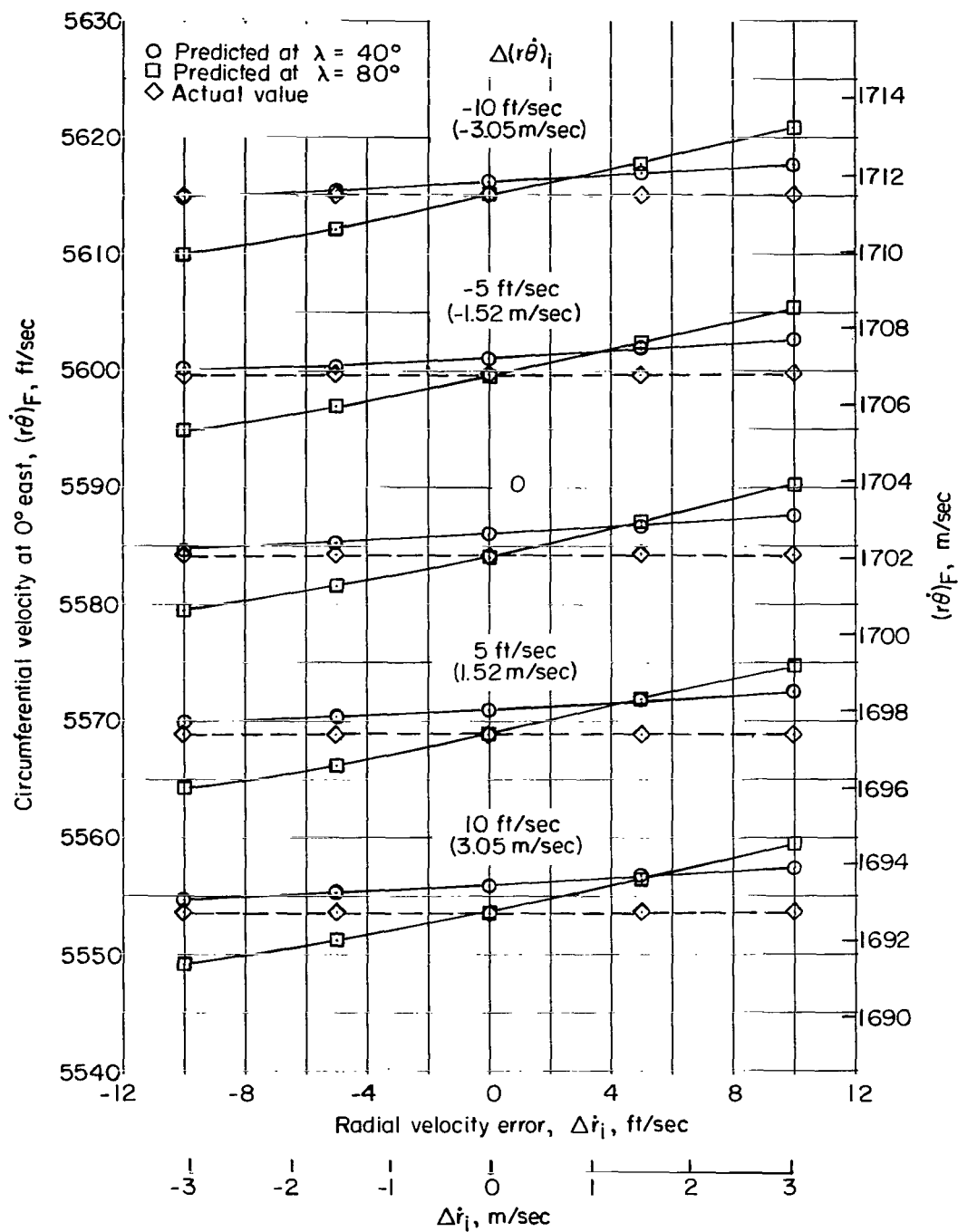
(c) Command module in 81-nautical-mile (150.01-kilometer) altitude circular orbit ( $\Delta h_0 = 1$  nautical mile or 1.852 kilometers).

Figure 5.- Concluded.



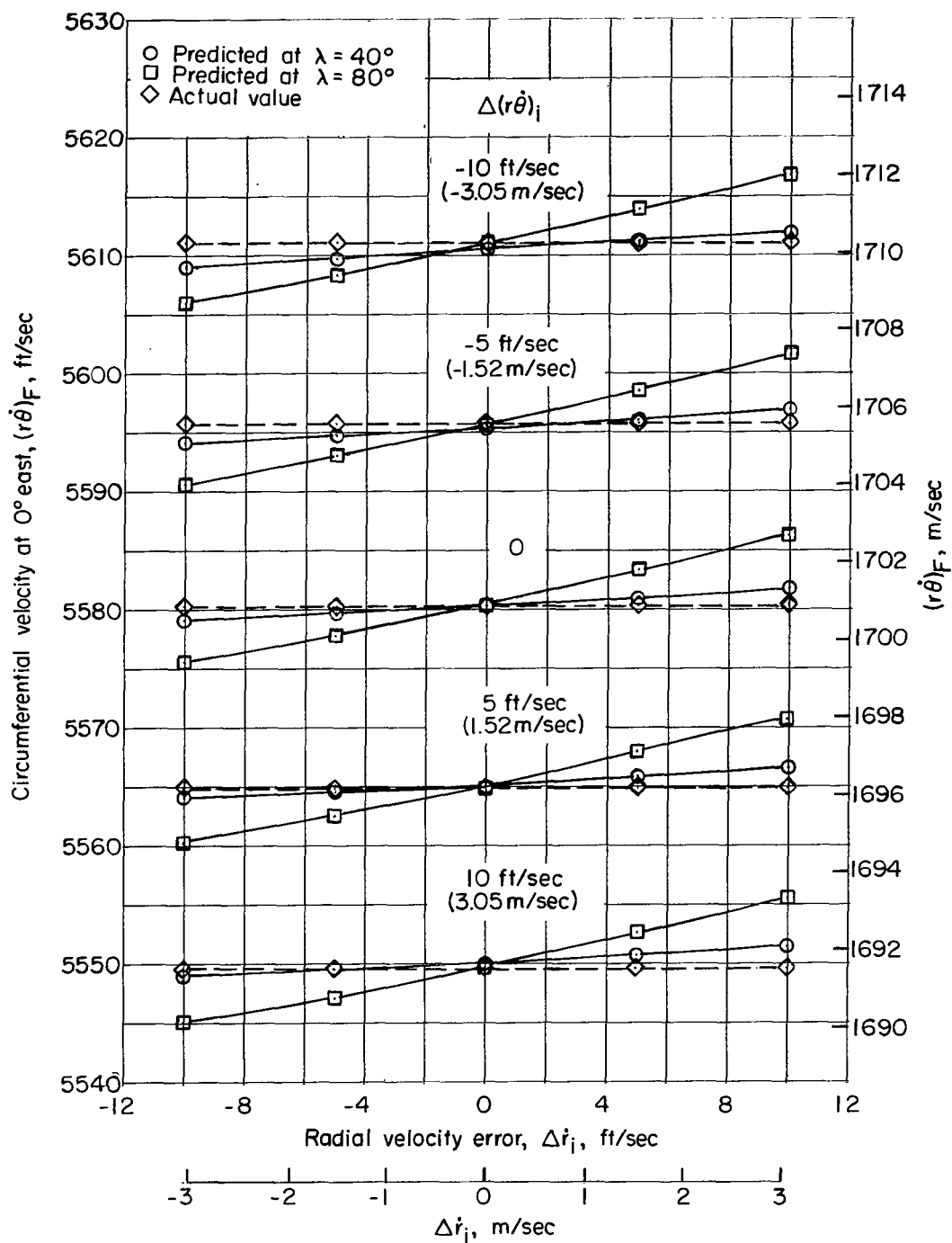
(a) Command module in 80-nautical-mile (148.16-kilometer) altitude circular orbit ( $\Delta h_0 = 0$ ).

Figure 6.- Circumferential velocity at point of nominal powered-descent initiation ( $0^\circ$  East longitude) for various errors in velocity at Hohmann insertion.



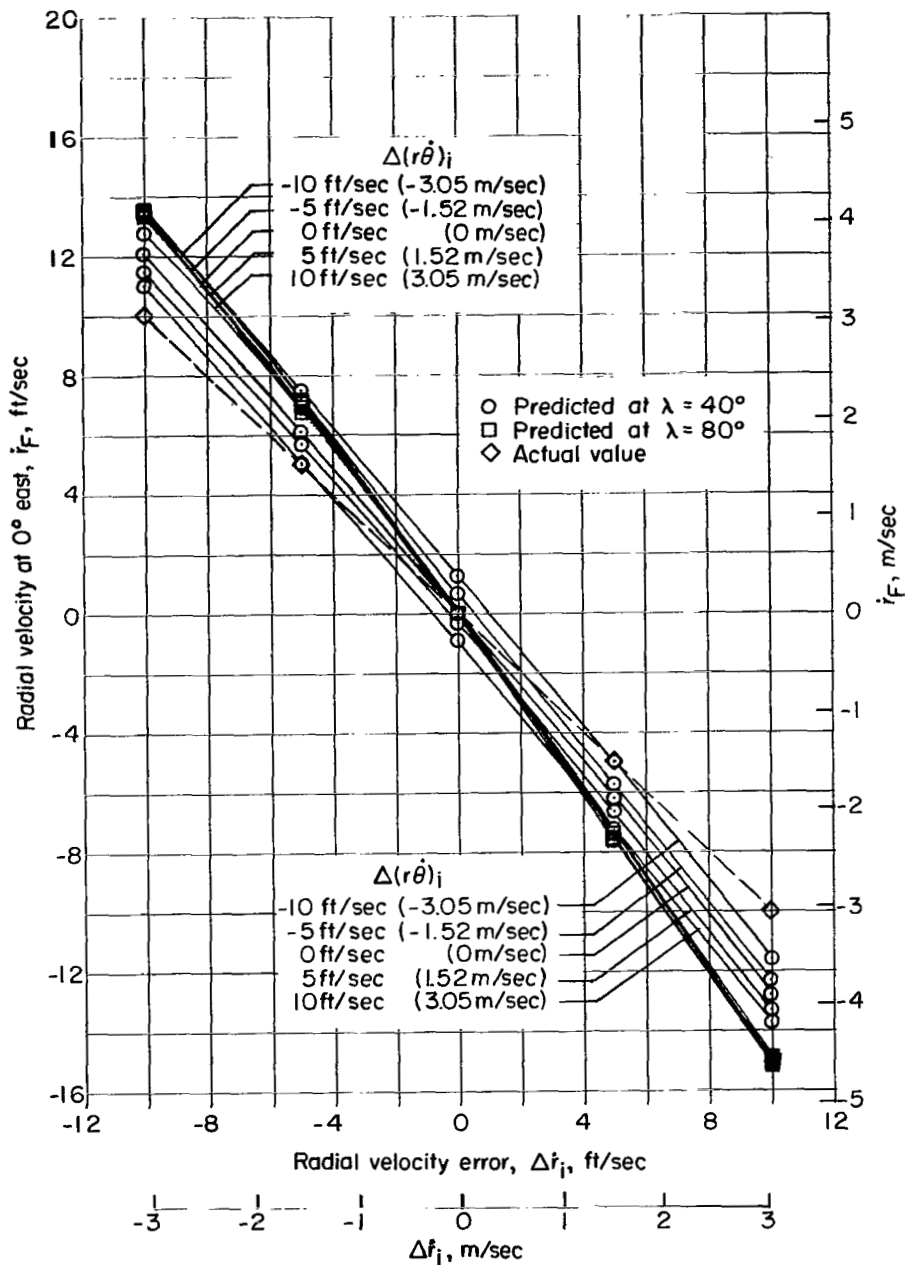
(b) Command module in 79-nautical-mile (146.31-kilometer) altitude circular orbit ( $\Delta h_0 = -1$  nautical mile or -1.852 kilometers).

Figure 6.- Continued.



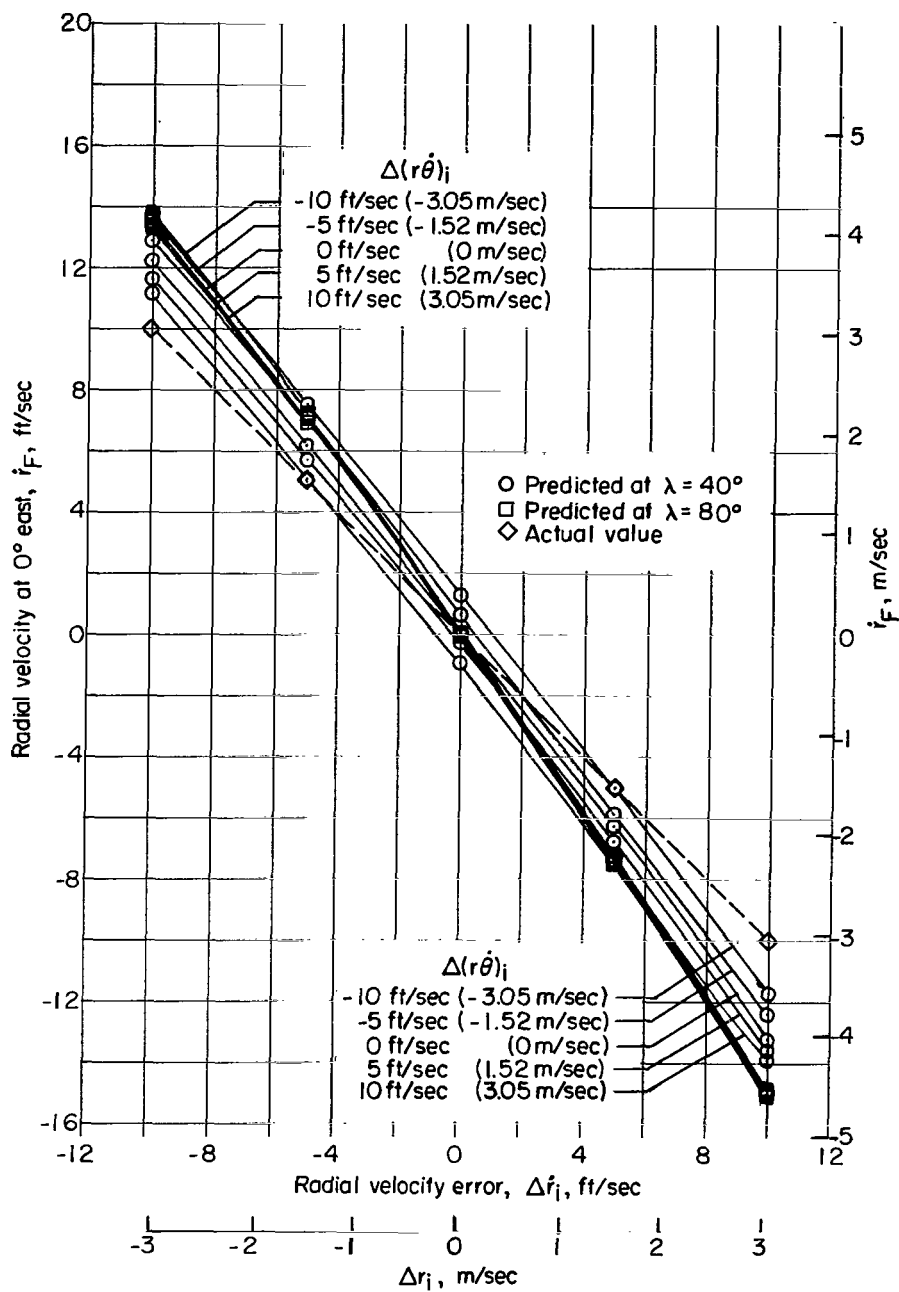
(c) Command module in 81-nautical-mile (150.01-kilometer) altitude circular orbit ( $\Delta h_0 = 1$  nautical mile or 1.852 kilometers).

Figure 6.- Concluded.



(a) Command module in 80-nautical-mile (148.16-kilometer) altitude circular orbit ( $\Delta h_0 = 0$ ).

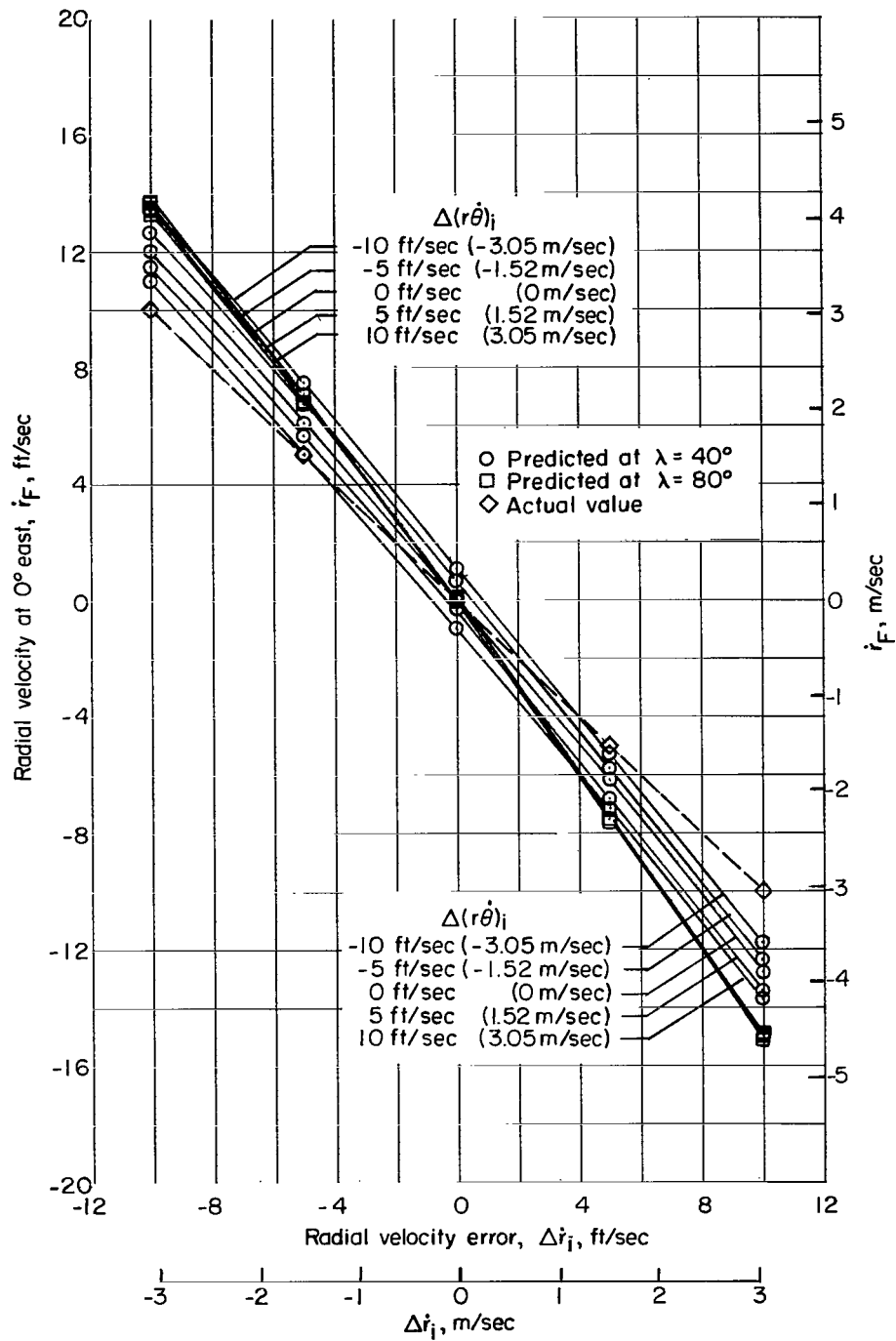
Figure 7.- Radial velocity at point of nominal powered-descent initiation (0° East longitude) for various errors in velocity at Hohmann insertion.



(b) Command module in 79-nautical-mile (146.31-kilometer) altitude circular orbit ( $\Delta h_0 = -1$  nautical mile or -1.852 kilometers).

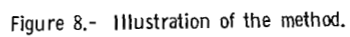
Figure 7.- Continued.

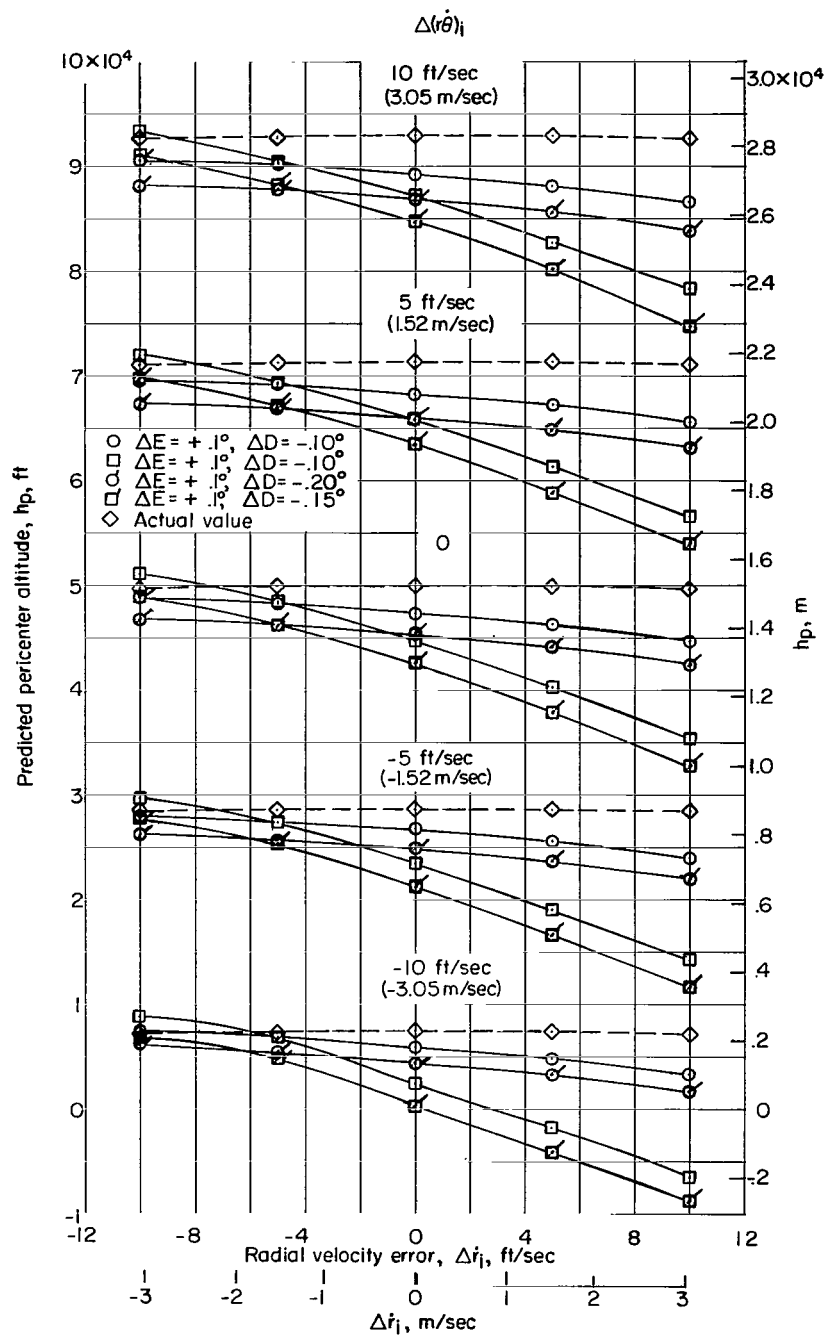




(c) Command module in 81-nautical-mile (150.01-kilometer) altitude circular orbit ( $\Delta h_0 = 1$  nautical mile or 1.852 kilometers).

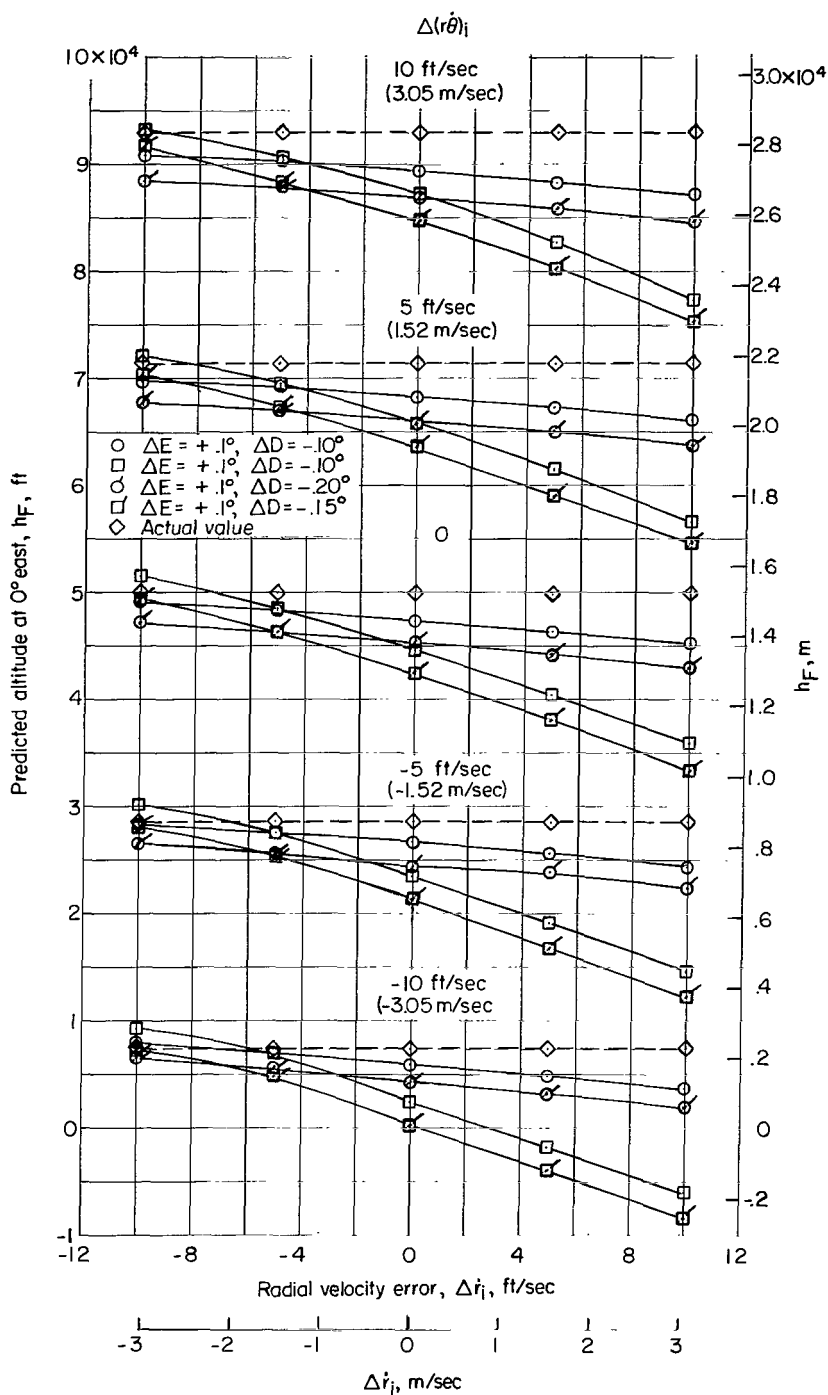
Figure 7.- Concluded.





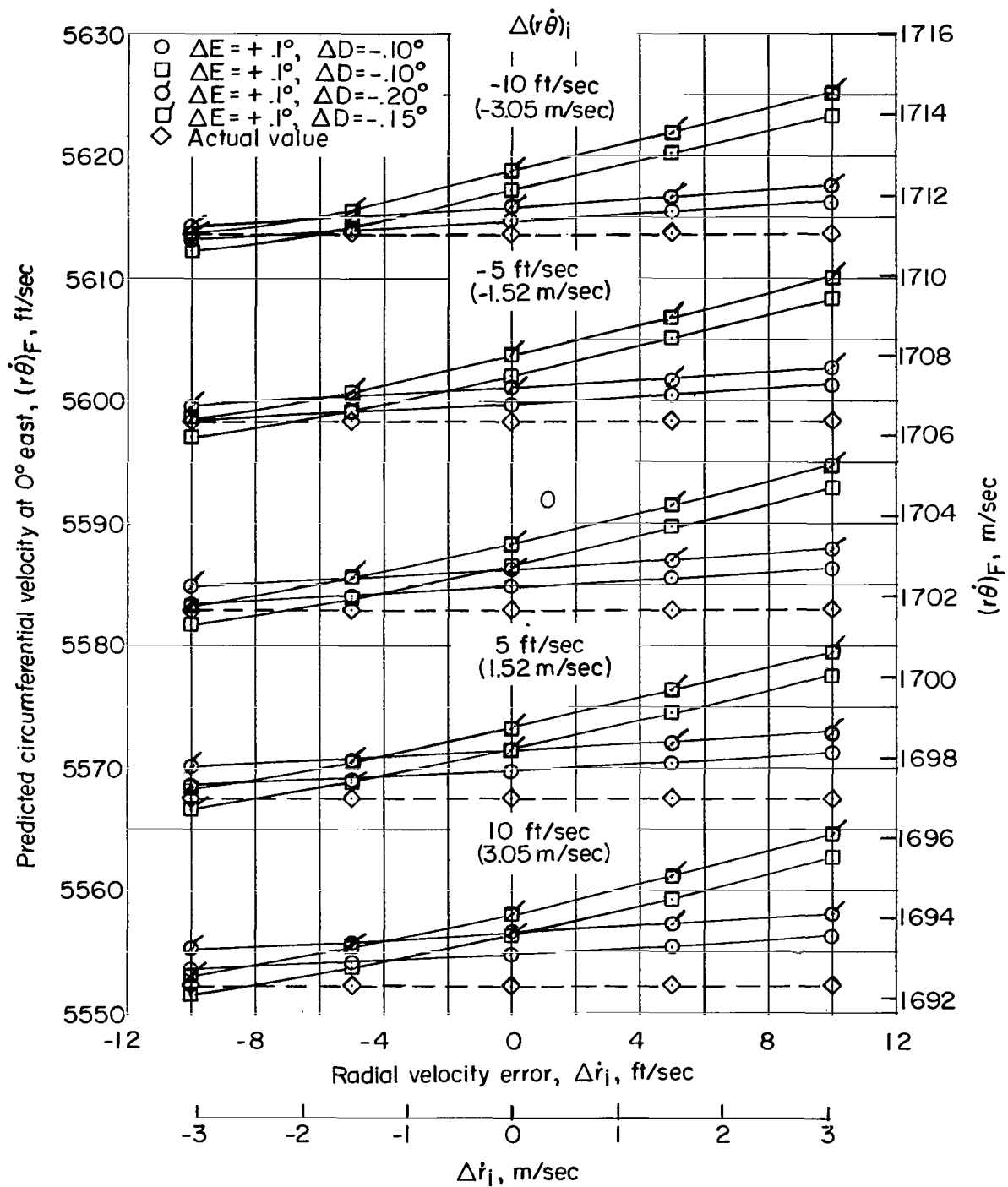
(a) Predicted pericynthion altitude.

Figure 9.- Predicted orbital parameters for that combination of measurement uncertainties ( $+\Delta E$ ,  $-\Delta D$ ) which results in the lowest predicted pericenter altitudes, for various errors at Hohmann insertion from an 80-nautical-mile (148.16-kilometer) altitude circular orbit. (Circles and squares both flagged and unflagged denote measurements made at  $\lambda$  equals  $40^\circ$  and  $80^\circ$ , respectively.)



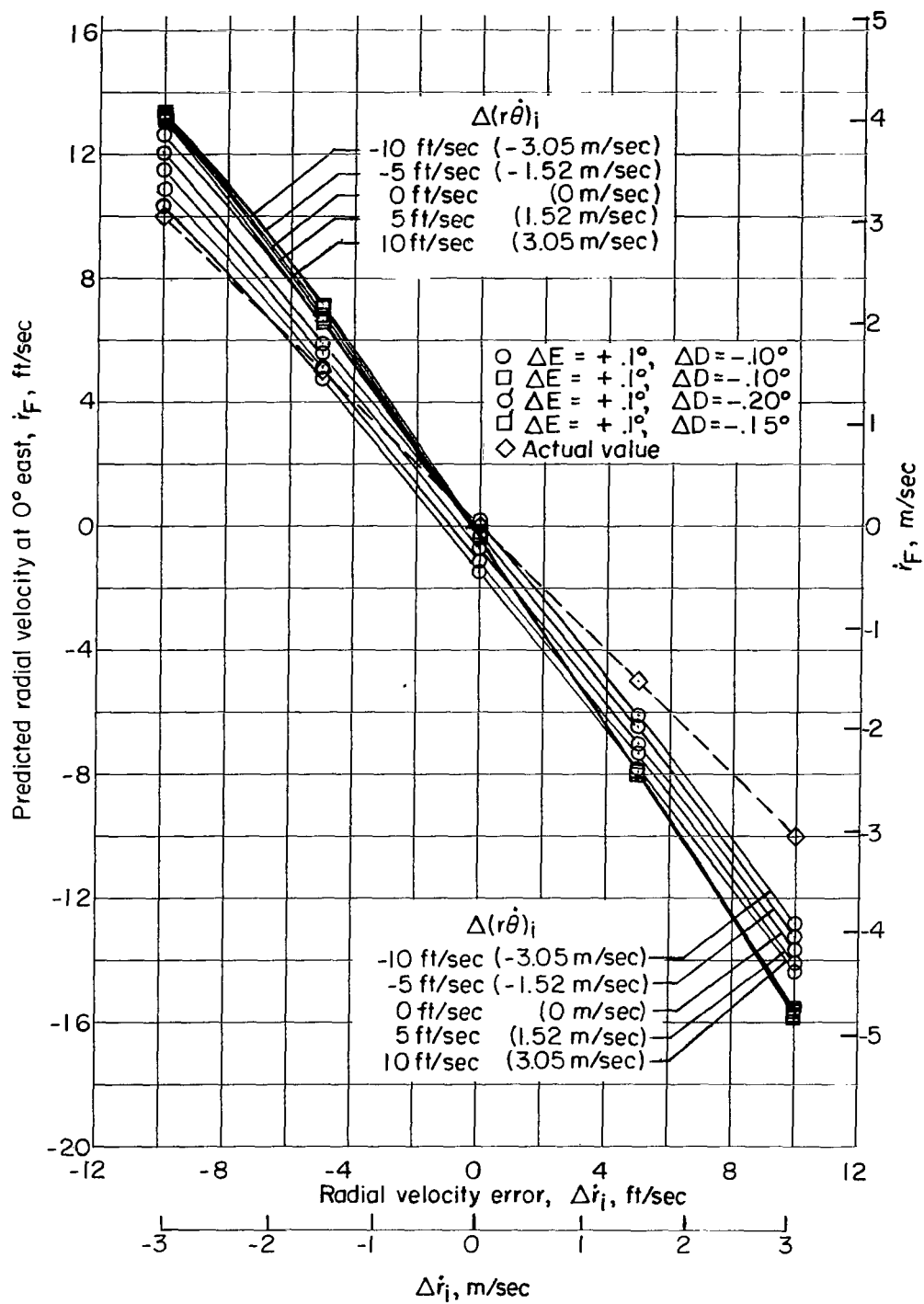
(b) Predicted altitude at nominal point of powered-descent initiation, 0° East.

Figure 9.- Continued.



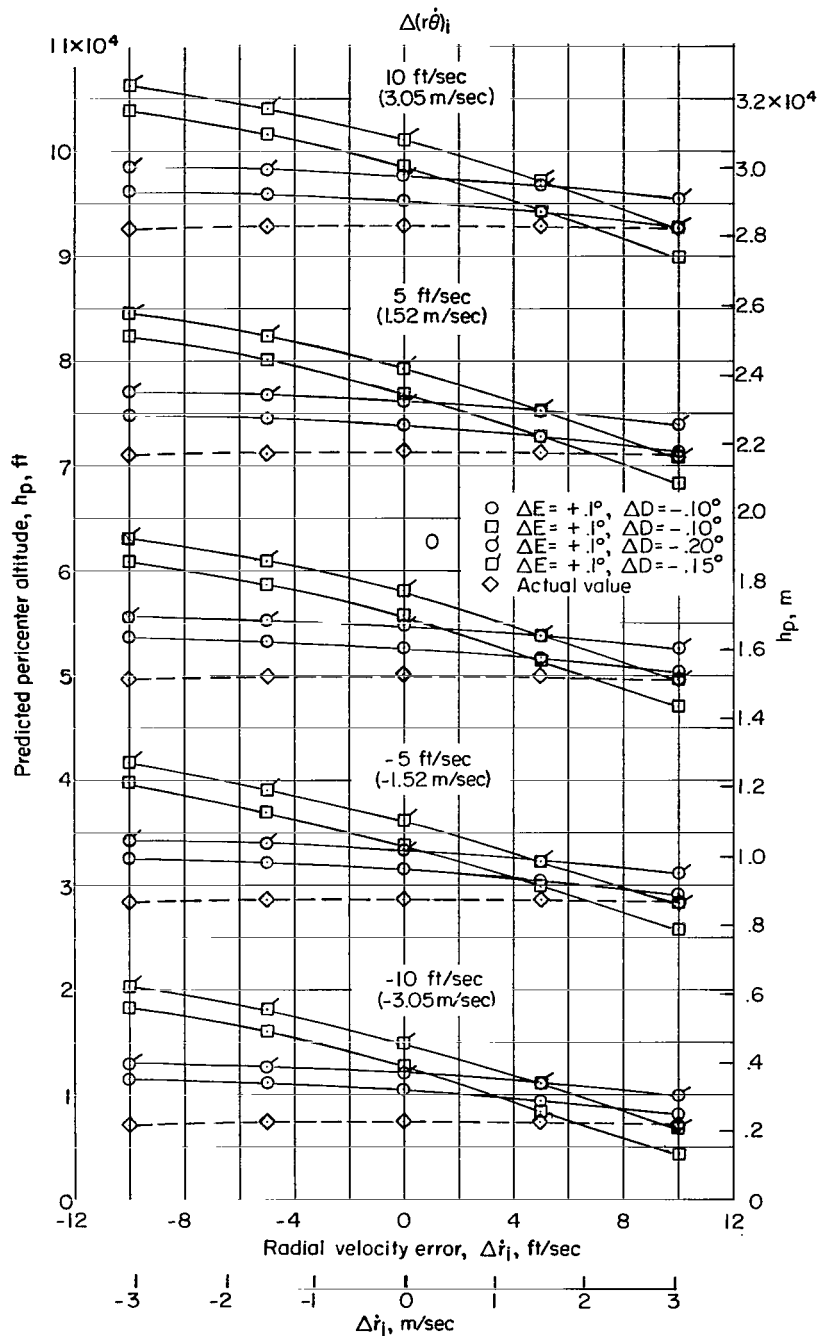
(c) Predicted circumferential velocity at nominal point of powered-descent initiation,  $0^\circ$  East.

Figure 9.- Continued.



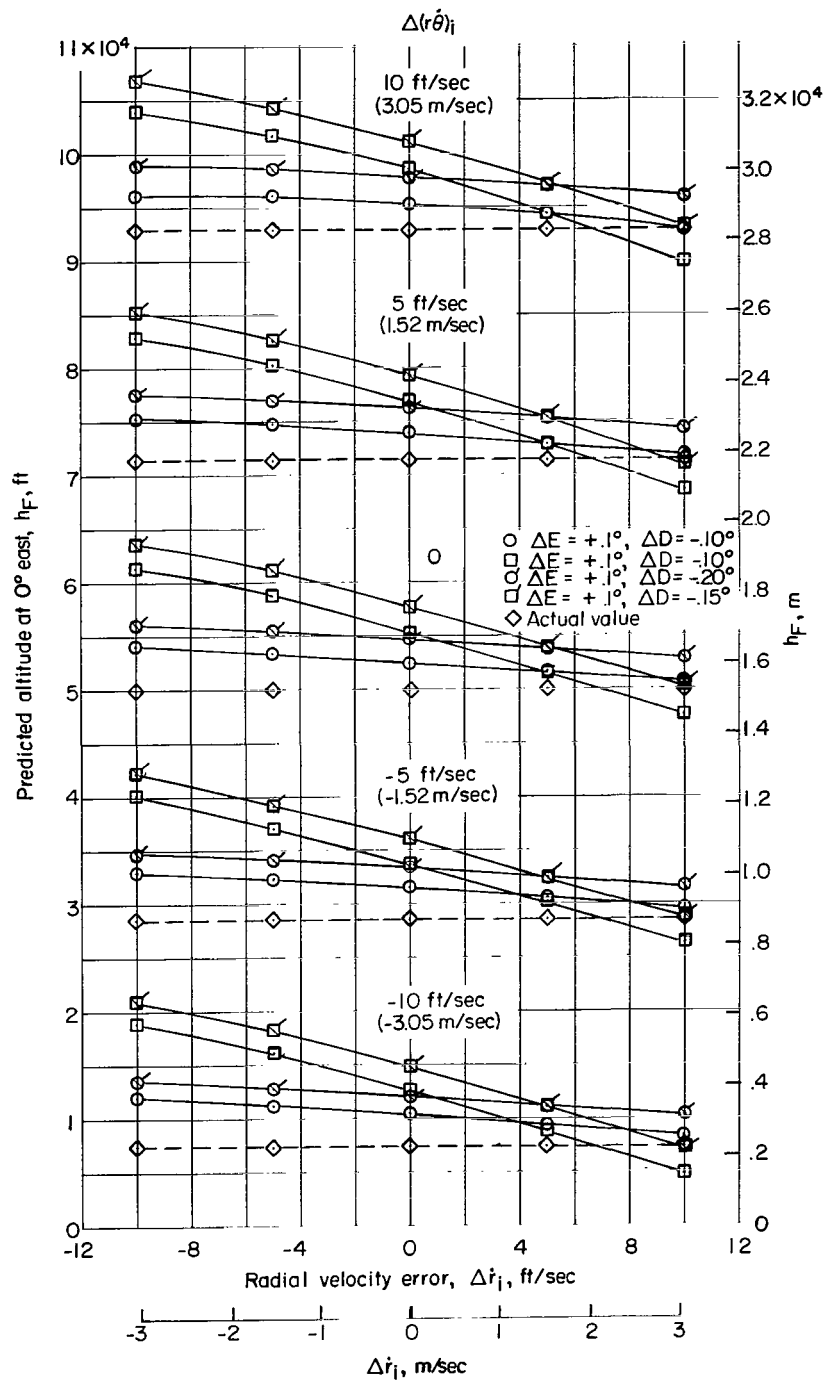
(d) Predicted radial velocity at nominal point of powered-descent initiation, 0° East. (Note that flagged and unflagged symbols coincide.)

Figure 9.- Concluded.



(a) Predicted pericynthion altitude.

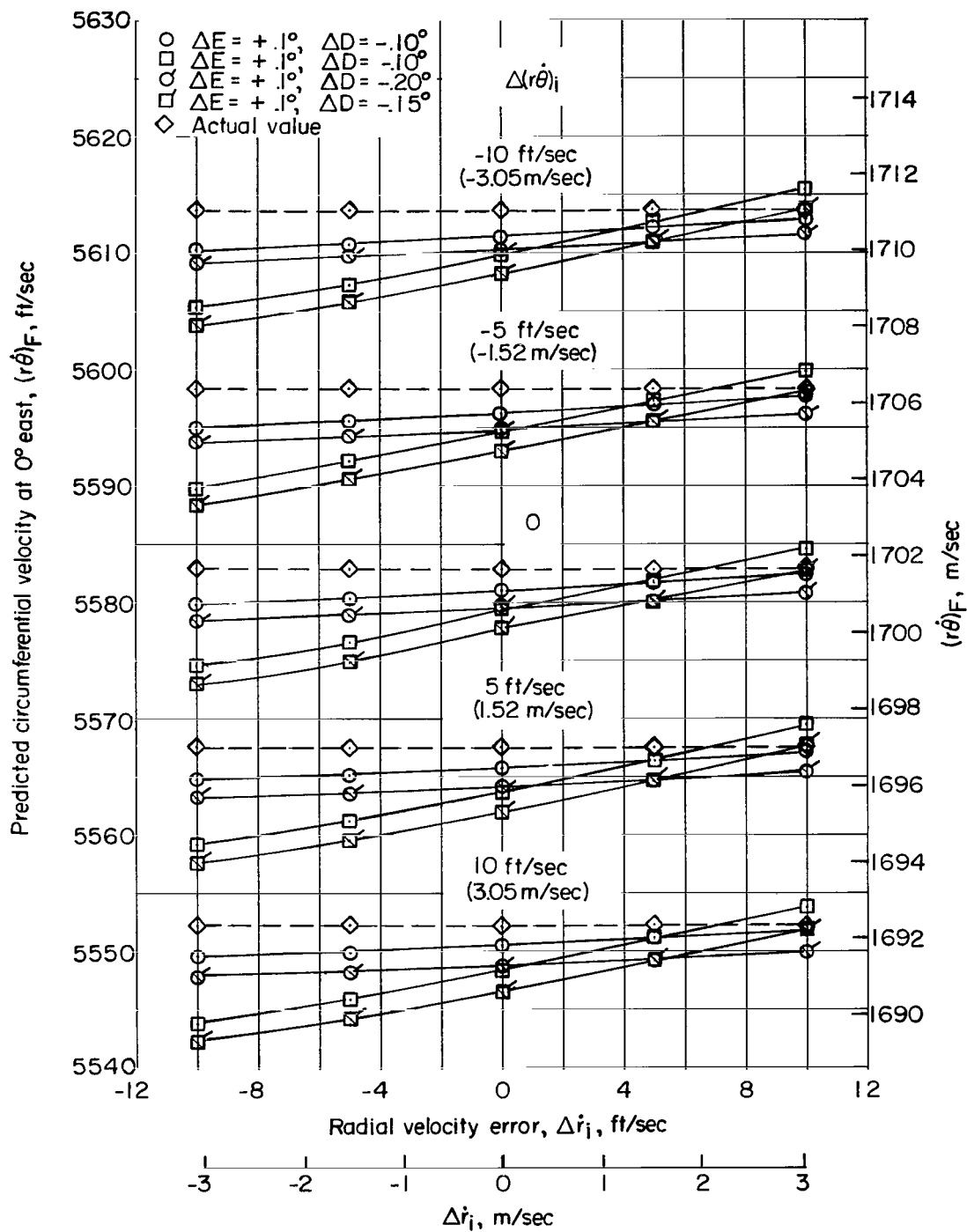
Figure 10.- Predicted orbital parameters for that combination of measurement uncertainties ( $-\Delta E$ ,  $+\Delta D$ ) which results in the highest predicted pericenter altitudes, for various errors at Hohmann insertion from an 80-nautical-mile (148.16-kilometer) altitude circular orbit. (Circles and squares both flagged and unflagged denote measurements made at  $\lambda$  equals  $40^\circ$  and  $80^\circ$ , respectively.)



(b) Predicted altitude at nominal point of powered-descent initiation, 0° East.

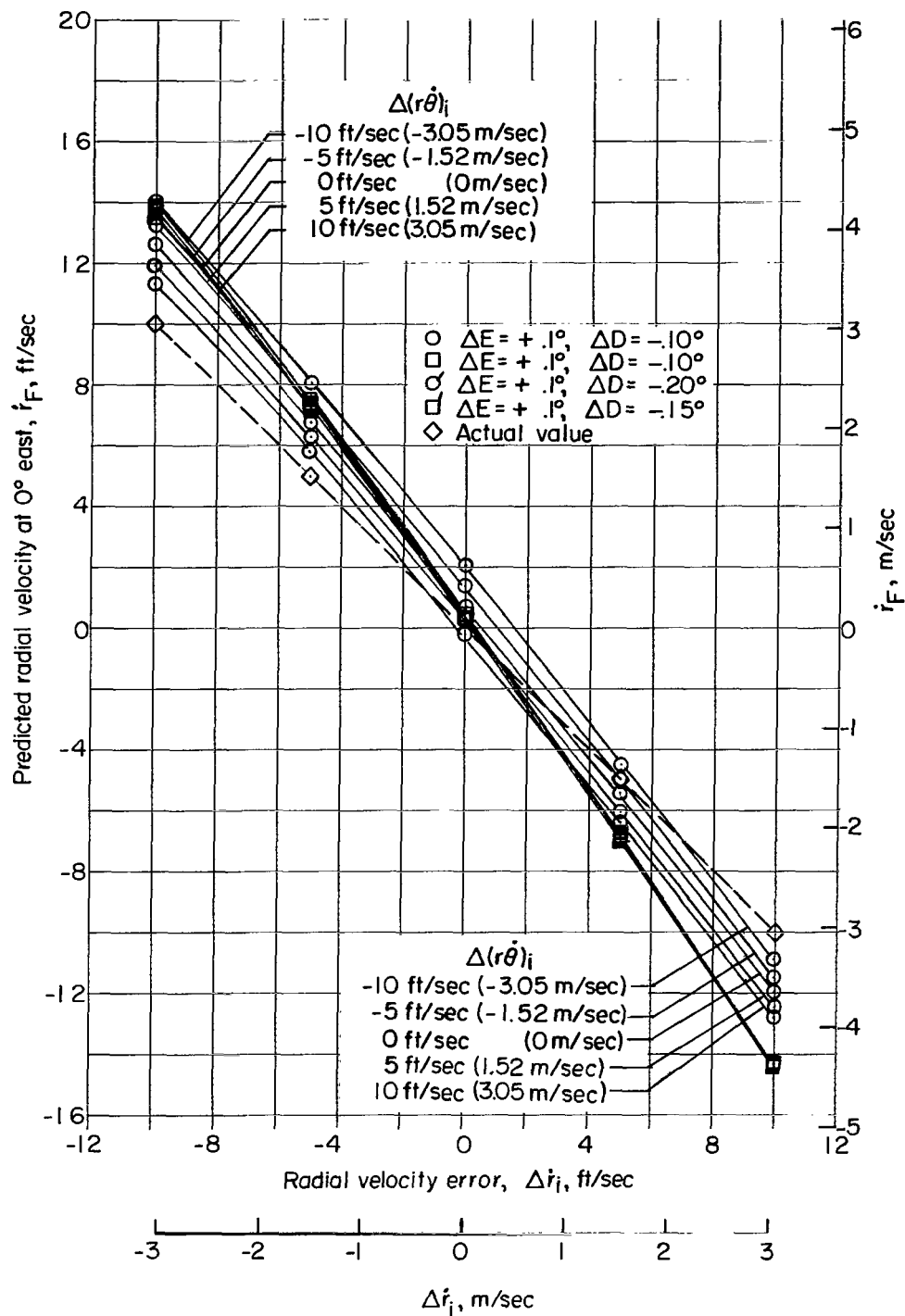
Figure 10.- Continued.





(c) Predicted circumferential velocity at nominal point of powered-descent initiation,  $0^\circ$  East.

Figure 10.- Continued.



(d) Predicted radial velocity at nominal point of powered-descent initiation, 0° East. (Note that flagged and unflagged symbols coincide.)

Figure 10.- Concluded.

**FIRST CLASS MAIL**

050 011 46 51 305 00226 CCSC3  
AIR FORCE STAGS GENERAL BYVALI /  
KIRILLOV AIR FORCE 050 011 46 51 305 00226

ALL THE ABOVE INFORMATION IS TRUE AND CORRECT TO THE BEST OF MY KNOWLEDGE.

**POSTMASTER:** If Undeliverable (Section 158  
Postal Manual) Do Not Return

*"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

## NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

**SPECIAL PUBLICATIONS:** Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

## TECHNOLOGY UTILIZATION

**PUBLICATIONS:** Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include *Tech Briefs*, Technology Utilization Reports and Notes, and Technology Surveys.

**CONTRACTOR REPORTS:** Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

*Details on the availability of these publications may be obtained from:*

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
Washington, D.C. 20546